SHORT PAPER PCB 7-2006

OBLIQUE COLLISIONS

ENGINEERING EQUATIONS, INPUT DATA AND MARC 1 APPLICATIONS

By:

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PC-BRAKE, INC.
2006

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PURPOSE OF PCB SHORT PAPERS

To provide the accident reconstruction practitioner with a concise discussion of the engineering equations and limiting factors involved, evaluation of critical input data, and the analysis of actual cases by use of the MARC 1 computer software.

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We hope that our Short Papers will assist the practitioner in better understanding the limitations inherent in any derivation of engineering equations, to properly use critical input data, to more accurately and effectively formulate his or her case under consideration, to become a better prepared expert in the field of accident reconstruction, and to more effectively utilize the full potential of the MARC 1 computer program.

Comments and suggestions are always invited by visiting our Discussion Forum and/or by writing to:

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Throughout the Short Papers we will extensively reference the 5th Edition of "Motor Vehicle Accident Reconstructions and Cause Analysis" by Rudolf Limpert, the "Accident Reconstruction Catalog" (ARC) CD, as well as the MARC 1 software.

OBLIQUE COLLISIONS

Part Three

Linear and Rotational Momentum
Only Angles Rotated After-Impact Known

1. DEFINITION OF LINEAR AND ROTATIONAL MOMENTUM

The basic formulation of linear and rotational momentum is discussed in Section 33-5 of the Text. The after-impact center-of-gravity velocities and angular velocities of each vehicle are calculated for known impact velocities and collision configuration by Equations 33-38 through 33-43.

The impulse components are calculated by Equations 33-35 and 33-36 based upon the relative velocities before as well as the center-of-gravity distances relative to the contact point (common velocity) between the vehicles.

As Example 33-6 of the Text shows, the analysis presented in Section 33-5(b) calculates the center-of-gravity velocities and angular velocities of both vehicles after impact based upon known data at the moment of impact.

It became desirable to solve the given system of equations for the velocities at the moment of impact in terms of the after-impact angular velocities, and hence, after-impact accident scene data (angles rotated and rotational tire-road friction coefficients).

2. WHAT ENGINEERING PRINCIPLES APPLY

For the discussion that follows we assume that the impact configuration is known. This means that the distances (or lever arms of the moment of momentum) are known from the collision diagram (MARC 1 – Y). With the masses and mass moment of inertia for both vehicles known, a-, b- and c-values are also known. See Figure 33-14 of the Text for a collision diagram example. We also assume that the approach angles are known for both vehicles.

Inspection of Equations 33-35 and 33-36 reveals that the impulses in the x- and y-directions are only a function of the impact velocity vectors V_{11} and V_{21}. Inspection of the right-hand sides of Equations 33-40 and 33-43 likewise shows that the after-impact angular velocity of each vehicle is only a function of the impact velocities V_{11} and V_{21} both in terms of magnitudes and approach angles. Since the approach angles are assumed to be known, we have only the impact velocity magnitudes as unknowns. If both after-impact angular velocities are known for each vehicle from accident scene data (after-impact angle rotated and rotational tire/road friction coefficient), we have two independent equations with only two unknowns. Solving Equations 33-40 and 33-43 for the impact velocities of each vehicle results in the equation presented on the following pages. See Example 21-4 (page 281) of the Text for a discussion on rotational coefficient of friction and angular velocity calculated from after-impact angle rotated.

The resulting equations are shown on the pages that follow and are the basis of MARC 1-X 7.

3.0 NON-CENTRAL OBLIQUE IMPACT

We will analyze the crash test discussed in PCB 5 – 2006, Section 4.1. A stationary VW Derby was impacted by VW Golf 1 at 29 mph. Standard linear momentum was used to reconstruct the crash test, resulting in a predicted impact velocity of approximately 27 to 28 mph. Using MARC 1 – X6 resulted in impact velocities of 28 to 29 mph.

Since MARC 1 – X 7 requires collision configuration data, the vehicle contact diagram is established by use of MARC 1 –Y shown PCB 6 – 2006, RUN 1. The vehicle dimensions were obtained from published data.

All input data are shown in MARC 1 – X7, RUN 1. The impact velocities are 30.96 mph for the Golf and 3.86 mph for the Derby. Inspection of RUN 1 indicates 7 degrees of after-impact rotation for the Golf. This value was obtained from Run-Out and Rest Position Diagram (Figure 4 of PCB 5-2006). The after-impact departure angle of the Golf is not identical with the angle rotated after impact. Since the Derby was stationary at impact, no after-impact rotation of the Golf exists. Using an after-impact rotation of 0.1 degrees, or approximately zero, the impact velocities become 29.68 and -0.08 mph, respectively as shown in MARC 1 – X7, RUN 2.

If this had been an actual accident, and no information had been available concerning the velocity of the Derby at impact, the reconstructionist can only conclude that the Golf traveled at a velocity of approximately 29 to 31 mph, with the Derby traveling at a low velocity less than 5 mph, or was possibly stopped at the moment of impact.

The after-impact data of the Derby are a more accurate indicator of the impact velocity of the Golf due to its significant rotation after-impact when compared with the Golf.

The impact velocities obtained from MARC 1 – X7 can be used in MARC 1 – X8, the “crash test” software program, to determine all essential after-impact data from the impact velocity ranges established by MARC 1 – X7. Secondary accident data obtained from witness statements and other non-objective data may be used to establish more probable impact velocities within the range established by MARC 1 – X7. The subject of determining “missing” accident data will be discussed in Short Paper PCB 8 – 2006.

Impact Speed Based on After Impact Angles of Rotation

Impact speed of V1

$$V_{11} = \frac{(\omega_{12} - \omega_{11})N - (\omega_{22} - \omega_{21})K}{MN - KP} \text{ ; ft/sec}$$

Impact speed of V2

$$V_{11} = \frac{(\omega_{12} - \omega_{11})P - (\omega_{22} - \omega_{21})M}{KP - MN} \text{ ; ft/sec}$$

Where:

$K = L_{1xc}\cos\alpha_{21} + L_{1xb}\sin\alpha_{21} - L_{1ya}\cos\alpha_{21} - L_{1yc}\sin\alpha_{21}$; 1/ft

$M = L_{1ya}\cos\alpha_{11} + L_{1yc}\sin\alpha_{11} - L_{1xc}\cos\alpha_{11} - L_{1xb}\sin\alpha_{11}$; 1/ft

$N = L_{2ya}\cos\alpha_{21} + L_{2yc}\sin\alpha_{21} - L_{2xc}\cos\alpha_{21} - L_{2xb}\sin\alpha_{21}$; 1/ft

$P = L_{2xc}\cos\alpha_{11} + L_{2xb}\sin\alpha_{11} - L_{2ya}\cos\alpha_{11} - L_{2yc}\sin\alpha_{11}$; 1/ft

$$L_{1xc} = \frac{cl_{1x}}{I_1(ab - c^2)} \text{ ; 1/ft}$$

$$L_{1xb} = \frac{bl_{1x}}{I_1(ab - c^2)} \text{ ; 1/ft}$$

$$L_{1ya} = \frac{al_{1y}}{I_1(ab - c^2)} \text{ ; 1/ft}$$

$$L_{1yc} = \frac{cl_{1y}}{I_1(ab - c^2)} \text{ ; 1/ft}$$

$$L_{2ya} = \frac{al_{2y}}{I_2(ab - c^2)} \text{ ; 1/ft}$$

Impact Speed Based on After Impact Angles of Rotation

$$L_{2yc} = \frac{c l_{2y}}{I_2(ab - c^2)} \quad ; 1/\text{ft}$$

$$L_{2xc} = \frac{c l_{2x}}{I_2(ab - c^2)} \quad ; 1/\text{ft}$$

$$L_{2xb} = \frac{b l_{2x}}{I_2(ab - c^2)} \quad ; 1/\text{ft}$$

$$a = \frac{1}{m_1} + \frac{1}{m_2} + \frac{l_{1x}^2}{I_1} + \frac{l_{2x}^2}{I_2} \quad ; \text{ft/(lb sec}^2)$$

$$b = \frac{1}{m_1} + \frac{1}{m_2} + \frac{l_{1y}^2}{I_1} + \frac{l_{2y}^2}{I_2} \quad ; \text{ft/(lb sec}^2)$$

$$c = \frac{l_{1x} l_{1y}}{I_1} + \frac{l_{2x} l_{2y}}{I_2} \quad ; \text{ft/(lb sec}^2)$$

Angular velocity after impact:

$$\omega_{i2} = \sqrt{\frac{W_i f_{Ri} L_i \beta_i}{I_i}} \quad ; \text{rad/sec}$$

ω_{i1} = angular velocity before impact (if any) determined from steering inputs, road curvature, etc, is small compared to after impact angular velocities and can be ignored in most cases.

Friday, March 17, 2006

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS
****** PROGRAM 'X-7' RUN FOR PCB 7 - 2006, X7 - RUN 1 ******
ANGLES ROTATED/2 VEHICLES

Information For Vehicles		1993 VW GOLF	1993 VW DERBY
Vehicle Weight, LBS:	===>	1918.00	2007.00
Vehicle Wheelbase, FT:	===>	8.12	7.70
Vehicle Length, FT:	===>	13.10	12.40
Distance from Center of Gravity to Contact Point:			
Along the X-Axis, FT:	===>	0.83	-2.75
Along the Y-Axis, FT:	===>	5.00	-2.00
Initial Angular Velocity, DEG/SEC:	===>	0.00	0.00
Approach Angle, DEG:	===>	90.00	180.00
Surface #1			
Pre-Impact Braking Distance, FT:	===>	0.00	0.00
Pre-Impact Deceleration, g-UNITS:	===>	0.00	0.00
Surface #2			
Pre-Impact Braking Distance, FT:	===>	0.00	0.00
Pre-Impact Deceleration, g-UNITS:	===>	0.00	0.00
Rotation After Impact, DEG:	===>	7.00	123.00
Direction of Rotation, Vehicle 1:	===>	Counterclockwise	
Direction of Rotation, Vehicle 2:	========>		Clockwise
After-Impact Coefficient of Rotational Friction, D'LESS:	===>	0.15	0.25

Mass Moment of Inertia, FT·LBS/SEC:²:	===>	816.09	766.51
Angular Velocity after Impact, DEG/SEC:	==>	33.88	-188.48
Speed at Impact, MPH:	===>	30.96	3.86
Pre-Impact Speed, MPH:	===>	30.96	3.86

Friday, March 17, 2006

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS
******* PROGRAM 'X-7' RUN FOR PCB 7 – 2006, X7, RUN 2 *******
ANGLES ROTATED/2 VEHICLES

Information For Vehicles		1993 VW GOLF	1993 VW DERBY
Vehicle Weight, LBS:	===>	1918.00	2007.00
Vehicle Wheelbase, FT:	===>	8.12	7.70
Vehicle Length, FT:	===>	13.10	12.40
Distance from Center of Gravity to Contact Point:			
Along the X-Axis, FT:	===>	0.83	-2.75
Along the Y-Axis, FT:	===>	5.00	-2.00
Initial Angular Velocity, DEG/SEC:	===>	0.00	0.00
Approach Angle, DEG:	===>	90.00	180.00
Surface #1			
Pre-Impact Braking Distance, FT:	===>	0.00	0.00
Pre-Impact Deceleration, g-UNITS:	===>	0.00	0.00
Surface #2			
Pre-Impact Braking Distance, FT:	===>	0.00	0.00
Pre-Impact Deceleration, g-UNITS:	===>	0.00	0.00
Rotation After Impact, DEG:	===>	0.10	123.00
Direction of Rotation, Vehicle 1:	===>	Counterclockwise	
Direction of Rotation, Vehicle 2:	======>		Clockwise
After-Impact Coefficient of Rotational Friction, D'LESS:	===>	0.15	0.25
Mass Moment of Inertia, FT·LBS/SEC:²:	===>	816.09	766.51
Angular Velocity after Impact, DEG/SEC:	==>	4.05	-188.48
Speed at Impact, MPH:	===>	29.68	-0.08
Pre-Impact Speed, MPH:	===>	29.68	0.08