

**SHORT PAPER PCB 6-2006**

# **OBLIQUE COLLISIONS**

**ENGINEERING EQUATIONS, INPUT DATA AND MARC 1 APPLICATIONS**

**By:**

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## PURPOSE OF PCB SHORT PAPERS

To provide the accident reconstruction practitioner with a concise discussion of the engineering equations and limiting factors involved, evaluation of critical input data, and the analysis of actual cases by use of the MARC 1 computer software.

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We hope that our Short Papers will assist the practitioner in better understanding the limitations inherent in any derivation of engineering equations, to properly use critical input data, to more accurately and effectively formulate his or her case under consideration, to become a better prepared expert in the field of accident reconstruction, and to more effectively utilize the full potential of the MARC 1 computer program.

Comments and suggestions are always invited by visiting our Discussion Forum and/or by writing to:

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Throughout the Short Papers we will extensively reference the 5<sup>th</sup> Edition of “Motor Vehicle Accident Reconstructions and Cause Analysis” by Rudolf Limpert, the “Accident Reconstruction Catalog” (ARC) CD, as well as the MARC 1 software.

# OBLIQUE COLLISIONS

## Part Two

### Linear and Rotational Momentum After-Impact Data of One Vehicle Missing

#### 1. DEFINITION OF LINEAR AND ROTATIONAL MOMENTUM

The basic formulation of linear and rotational momentum is discussed in Section 33-5 of the Text. The after-impact center-of-gravity velocities and angular velocities of each vehicle are calculated for known impact velocities and collision configuration by Equations 33-38 through 33-43.

The impulse components are calculated by Equations 33-35 and 33-36 based upon the relative velocities before as well as the center-of-gravity distances relative to the contact point (common velocity) between the vehicles.

As Example 33-6 of the Text shows, the analysis presented in Section 33-5(b) calculates the center-of-gravity velocities and angular velocities of both vehicles after impact based upon known data at the moment of impact.

It became desirable to solve the given system of equations for the velocities at the moment of impact in terms of the after-impact velocities, and hence, after-impact accident scene data.

#### 2. WHAT ENGINEERING PRINCIPLES APPLY

For the discussion that follows we assume that the impact configuration is known. This means that the distances (or lever arms of the moment of momentum) are known from the collision diagram (MARC 1 – Y). With the masses and mass moment of inertia for both vehicles known, a-, b- and c-values are also known. See Figure 33-14 of the Text for a collision diagram example. We also assume that the approach angles are known for both vehicles.

Inspection of Equations 33-35 and 33-36 reveals that the impulses in the x- and y-directions are only a function of the impact velocity vectors  $V_{11}$  and  $V_{21}$ . Inspection of the right-hand sides of Equations 33-38, 33-39, 33-41 and 33-42 likewise shows that the after-impact velocity components of each vehicle are only a function of the impact velocities  $V_{11}$  and  $V_{21}$  both in terms of magnitudes and approach angles. Since the approach angles are assumed to be known, we have only the impact velocity magnitudes as unknowns. If both after-impact velocity vectors (magnitudes and departure angles) are

known for each vehicle from accident scene data, we have four independent equations with only two unknowns. However, if the accident scene data provide after-impact information for one vehicle only, we can still solve for the four unknowns, namely impact velocities of both vehicles (two unknowns) and the departure velocity (magnitude and departure angle) of the unknown vehicle by use of the four independent equations developed from Equations 33-38, 33-39, 33-41 and 33-42.

The resulting equations are shown on the pages that follow and are the basis of MARC 1 – X6. MARC 1- X6 has the option of either having no after-impact data for Vehicle 1 or Vehicle 2. The two possible sets of impact velocities calculated for each vehicle, based upon after-impact data of either Vehicle 1 or Vehicle 2, are independent of each other, however, they rely upon the same impact configuration and accident scenario. If both after-impact accident scene data are accurate, both sets of impact velocities will be nearly identical, giving increased probability of one's accident reconstruction accuracy. If the after-impact data of one vehicle appear to be questionable, more weight should be given to the other impact velocity pair.

### 3.0 NON-CENTRAL OBLIQUE IMPACT

We will analyze the crash test discussed in PCB 5 – 2006, Section 4.1. A stationary VW Derby was impacted by VW Golf 1 at 29 mph. Standard linear momentum was used (Short Paper PCB 5 – 2006) to reconstruct the crash test, resulting in a predicted impact velocity of approximately 27 to 28 mph.

Since MARC 1 – X6 requires collision configuration data, the vehicle contact diagram is established by use of MARC 1–Y shown in PCB 6 – 2006, RUN1. The vehicle dimensions were obtained from published data.

All input data are shown for PROGRAM X, RUN 1. Use after-impact data of Vehicle 1 (Golf) results in 27.73 mph for the Golf and 0.04 mph, that is, stationary for the Derby. After-impact data of Vehicle 2 (Derby) yield 29.19 and 0.03 mph, respectively.

The reader is encouraged to conduct a parameter sensitivity study similar to the one described in Section 33-7 of the Text to determine how sensitive certain parameters are.

If, for example, no after-impact data had been available for Vehicle 1, Surface # 1 input data for Vehicle 1 would have been “0” (instead of “1”) in MARC 1- X9, RUN 1, resulting in 29.19 and 0.03 mph, respectively.

The “missing” after-impact data for Vehicle 1, as well as any other post crash data, can easily be determined by using MARC 1 – X 8, the “crash test” computer software based upon known impact velocities of both vehicles.

Impact speed of V1 based on V1 after-impact data:

$$V_{11-1} = \frac{(AD - BC) V_{12}}{D - B}; \text{ ft/sec}$$

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**Where:**

$V_{12}$  = velocity of V1 after impact, ft/sec

Impact speed of V2 based on V1 after-impact data:

$$V_{21-1} = \frac{(C - A) V_{12}}{D - B}; \text{ ft/sec}$$

---

**Where:**

$V_{12}$  = velocity of V1 after impact, ft/sec

Impact speed of V1 based on V2 after-impact data:

$$V_{11-2} = \frac{(EH - GF) V_{22}}{H - F}; \text{ ft/sec}$$

---

**Where:**

$V_{22}$  = velocity of V2 after impact, ft/sec

Impact speed of V2 based on V2 after-impact data:

$$V_{21-2} = \frac{(E - G) V_{22}}{H - F}; \text{ ft/sec}$$

---

**Where:**

$V_{22}$  = velocity of V2 after impact, ft/sec

$$b_i = \frac{b}{m_i(ab-c^2)}$$

$$c_i = \frac{c}{m_i(ab-c^2)}$$

$$a = \frac{1}{m_1} + \frac{1}{m_2} + \frac{l_{1x}^2}{I_1} + \frac{l_{2x}^2}{I_2} ; \text{ft}/(\text{lb sec}^2)$$

$$b = \frac{1}{m_1} + \frac{1}{m_2} + \frac{l_{1y}^2}{I_1} + \frac{l_{2y}^2}{I_2} ; \text{ft}/(\text{lb sec}^2)$$

$$c = \frac{l_{1x}l_{1y}}{I_1} + \frac{l_{2x}l_{2y}}{I_2} ; \text{ft}/(\text{lb sec}^2)$$

$$A = \frac{\cos \alpha_{12}}{\cos \alpha_{11} - a_i \cos \alpha_{11} - c_i \cos \alpha_{11}}$$

$$B = \frac{a_i \cos \alpha_{21} + c_i \sin \alpha_{21}}{\cos \alpha_{11} - a_i \cos \alpha_{11} - c_i \cos \alpha_{11}}$$

$$C = \frac{\sin \alpha_{12}}{\sin \alpha_{11} - c_i \cos \alpha_{11} - b_i \sin \alpha_{11}}$$

$$D = \frac{c_i \cos \alpha_{21} + b_i \sin \alpha_{21}}{\sin \alpha_{11} - c_i \cos \alpha_{11} - b_i \cos \alpha_{11}}$$

$$E = \frac{\cos \alpha_{22}}{a_i \cos \alpha_{11} + c_i \sin \alpha_{11}}$$

$$F = \frac{a_i \cos \alpha_{21} + c_i \sin \alpha_{21} - \cos \alpha_{21}}{a_i \cos \alpha_{11} + c_i \sin \alpha_{11}}$$

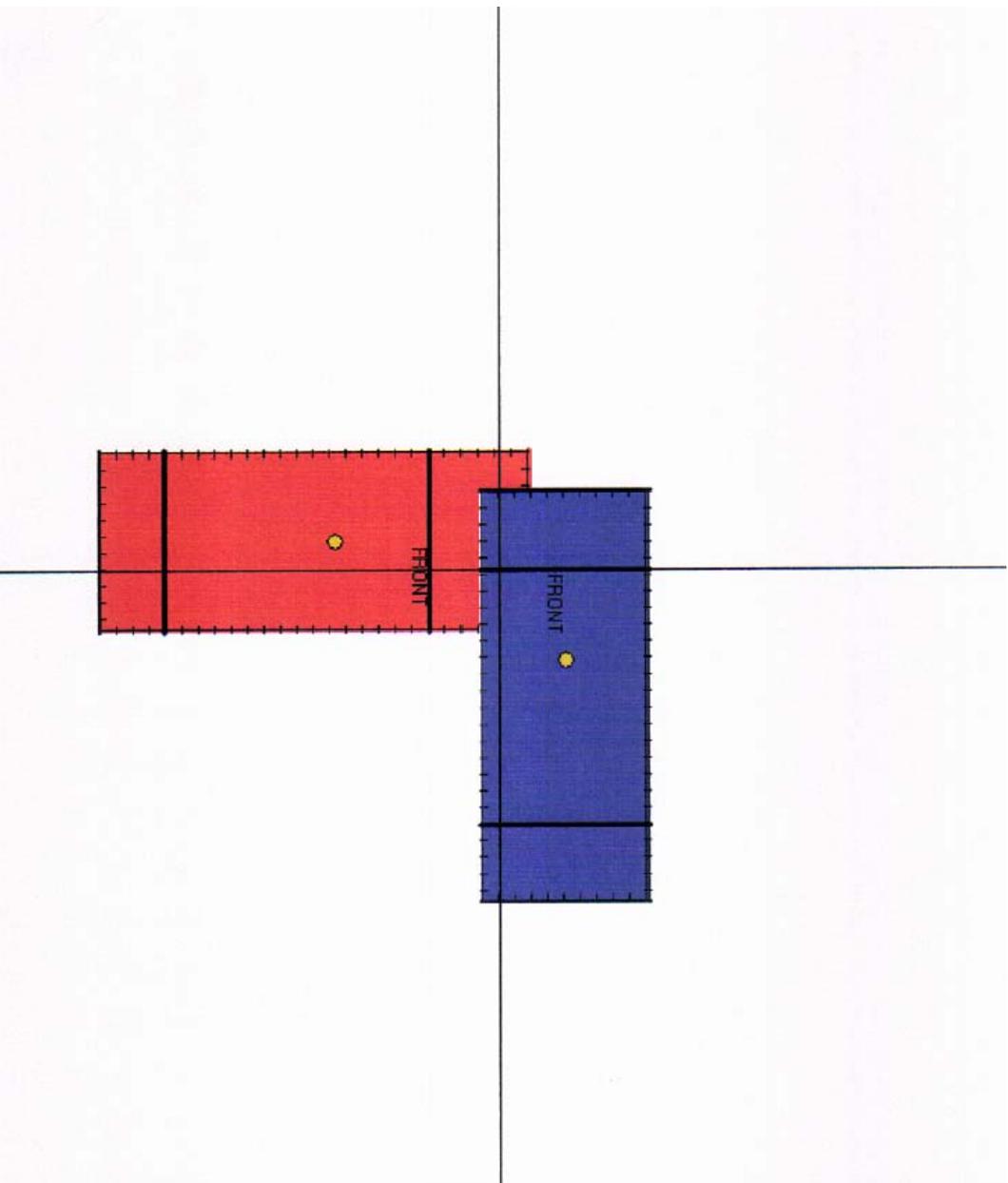
$$G = \frac{\sin \alpha_{22}}{c_i \cos \alpha_{11} + b_i \sin \alpha_{11}}$$

$$H = \frac{c_i \cos \alpha_{21} + b_i \sin \alpha_{21} - \sin \alpha_{21}}{c_i \cos \alpha_{11} + b_i \sin \alpha_{11}}$$

**Where:**

$$i = 1 \text{ or } i = 2$$

$$a_i = \frac{a}{m_i(ab-c^2)}$$



Caseld  
 pcb 6 2006, RUN 1

Vehicle Number **1** **2**

Width:   FT.

Length:   FT.

Wheelbase:   FT.

Bumper to front axle:   FT.

Percent weight on front axle:

Angle:   DEG.

Show Solid Car    Rotate Factor:  DEG.

Scalefactor:

Move Factor:  .05 FT.  .1 FT.  .2 FT.

Distance from Contact Point to C of G, FT:

Vehicle 1 (Red) X-dir: 0.85

Vehicle 1 (Red) Y-dir: 5.00

Vehicle 2 (Blue) X-dir: -2.75

Vehicle 2 (Blue) Y-dir: -2.00

Thursday, March 16, 2006

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS  
\*\*\*\*\* PROGRAM 'X-6' RUN FOR PCB 6 - 2006, X - 6, RUN 1 \*\*\*\*\*  
OBLIQUE COLLISION/LIMITED AFTER IMPACT DATA  
VEHICLE DATA

Information For Vehicles	1993 VW GOLF	1993 VW DERBY
Vehicle Weight, LBS:	====> 1918.00	2007.00
Vehicle Wheelbase, FT:	====> 8.12	7.70
Vehicle Length, FT:	====> 13.10	12.40
Mass Moment of Inertia, FT·LBS/SEC <sup>2</sup> :	====> 816.09	766.51
Distance from Center of Gravity to Contact Point:		
Along the X-Axis, FT:	====> 0.83	-2.75
Along the Y-Axis, FT:	====> 5.00	-2.00

NO BEFORE IMPACT SURFACE MEASUREMENTS

Surface #1

Distance Traveled After Impact, FT:	====> 10.20	9.80
After-Impact Deceleration, g-UNITS:	====> 0.90	0.45

Approach Angle, DEG:	====> 90.00	180.00
Departure Angle, DEG:	====> 97.00	80.00
Energy from Secondary Impacts, FT·LBS:	====> 0.00	0.00

OBLIQUE COLLISION/LINEAR MOMENTUM  
USING VEHICLE 1 DATA

Pre-Impact Speed, MPH:	====> 27.73	0.04
Speed at Impact, MPH:	====> 27.73	0.04
After-Impact Speed, MPH:	====> 16.60	11.50

OBLIQUE COLLISION/LINEAR MOMENTUM  
USING VEHICLE 2 DATA

Pre-Impact Speed, MPH:	====> 29.19	0.03
Speed at Impact, MPH:	====> 29.19	0.03
After-Impact Speed, MPH:	====> 16.60	11.50