SHORT PAPER PCB 1 - 2007

VELOCITY-TIME DIAGRAM

Its Effective Use in Accident Reconstruction and Court Room Presentation

By

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1.0 INTRODUCTION

In 1953, an excellent teacher introduced me to a clear and effective formulation of dynamic problems by use of the velocity-time diagram. Although things learned then were stashed away for nearly two decades, I thank Alfred Boege for teaching me the basics and getting me "hooked" on the desire to learn and to write.

When I started investigating and reconstructing traffic accidents and testifying in court, lawyers often wanted a simple "speed, time and distance" analysis done. Frequently, it did not involve much more than calculating skidding times or reaction distances. Those cases over 30 years ago were the beginning to develop and fine-tune the "velocity-time-diagram" method.

Several years ago while teaching one of my SAE International accident reconstruction seminars, during the first day we were discussing the maximum speed a vehicle could travel and still avoid the crash. The derivation of the motion equation is a relatively simple task once the velocity-time diagram is drawn and the area equations are written down. One seminar participant, an expert witness with a Ph.D. in mechanical engineering, remarked that this piece of information he just learned was worth all the money he paid for the entire 3-day seminar. I am certain the gentleman knew the information, yet never had it put together in such a simple demonstration.

The purposes of this paper are to review the velocity-time diagram fundamentals, to apply it to complicated two-vehicle motion analysis, to investigate electronic vehicle velocity recordings, and to demonstrate its effective use in courtroom and/or depositions. Example 20-2 of Reference 1 is a demonstration of an effective application of an actual case involving the velocity-time diagram in a trial in Hawaii.

2.0 FUNDAMENTALS OF THE VELOCITY-TIME DIAGRAM

The fundamentals of the velocity-time diagram are discussed in Section 20-1(b) of Reference 1, where several examples are shown in Section 20-1(g).

In the case of uniform velocity, the velocity of a vehicle remains constant regardless of time. The motion equation for calculating velocity V is:

$$V = S/t;\ \text{ft/sec} \qquad (1)$$

where: S = distance, ft
t = time, sec

Velocity is vector, that is, three physical measures are required to fully describe it. The measures are:

1. Line of action along which the velocity vector is acting. For example, the line of action may run north-south (or 90-270 degrees in the 360-degree coordinate system)
2. The direction in which the vehicle may travel. For example, north on the line of action, or at 90 degrees.
3. Magnitude, for example 60 ft/sec.

The 360-degree coordinate system automatically assigns the proper sign convention for the velocity. If the vehicle travels under 45 degrees, $\sin 45^o = 0.707$, while a vehicle traveling along the same line of action, however in the opposite direction under 225 degrees, $\sin 225^o = -0.707$.

Some times experts and others describe velocity measured in ft/sec as a vector, and speed in mph as a scalar. In momentum analysis, velocity is a vector. In speed, time and distance analysis velocity generally is treated as a scalar. In the velocity-time diagram analysis we are primarily concerned with the magnitude of velocity.

Equation 1 can be solved for distance as:

$$S = V\,t;\ \text{ft} \qquad (2)$$

In the velocity-time diagram we plot velocity V as a function of time as illustrated for the constant velocity motion in Figure 1. Since the velocity is constant, the velocity line runs parallel to the time axis. For example, for a car traveling at 60 ft/sec for 6 sec the car has traveled 360 ft. Inspection of Figure 1 reveals that the product of V times t equals the area under the velocity line, since the area of a rectangle equals base (time) times height (velocity). Consequently, we conclude that the area under the velocity line always equals the distance traveled during the time interval.

How do we use the velocity-time diagram in actual cases? The basic rules are always the same, involving the following four steps, namely (Reference 2):

1. Sketch the velocity-time diagram for the motion process under consideration. The final diagram may consist of the combination of two (or more) velocity-sub diagrams of each of the vehicles involved.

2. Write down the basic motion equations for the uniform acceleration/deceleration motion(s). Use the symbols used in the sketch.

3. Write down the distance equations while considering the particular areas involving additions and/or differences.

4. Develop a solution equation from a set of equations. One or more of the given equations always is the fundamental equation $a = V/t$, the other is a distance equation.

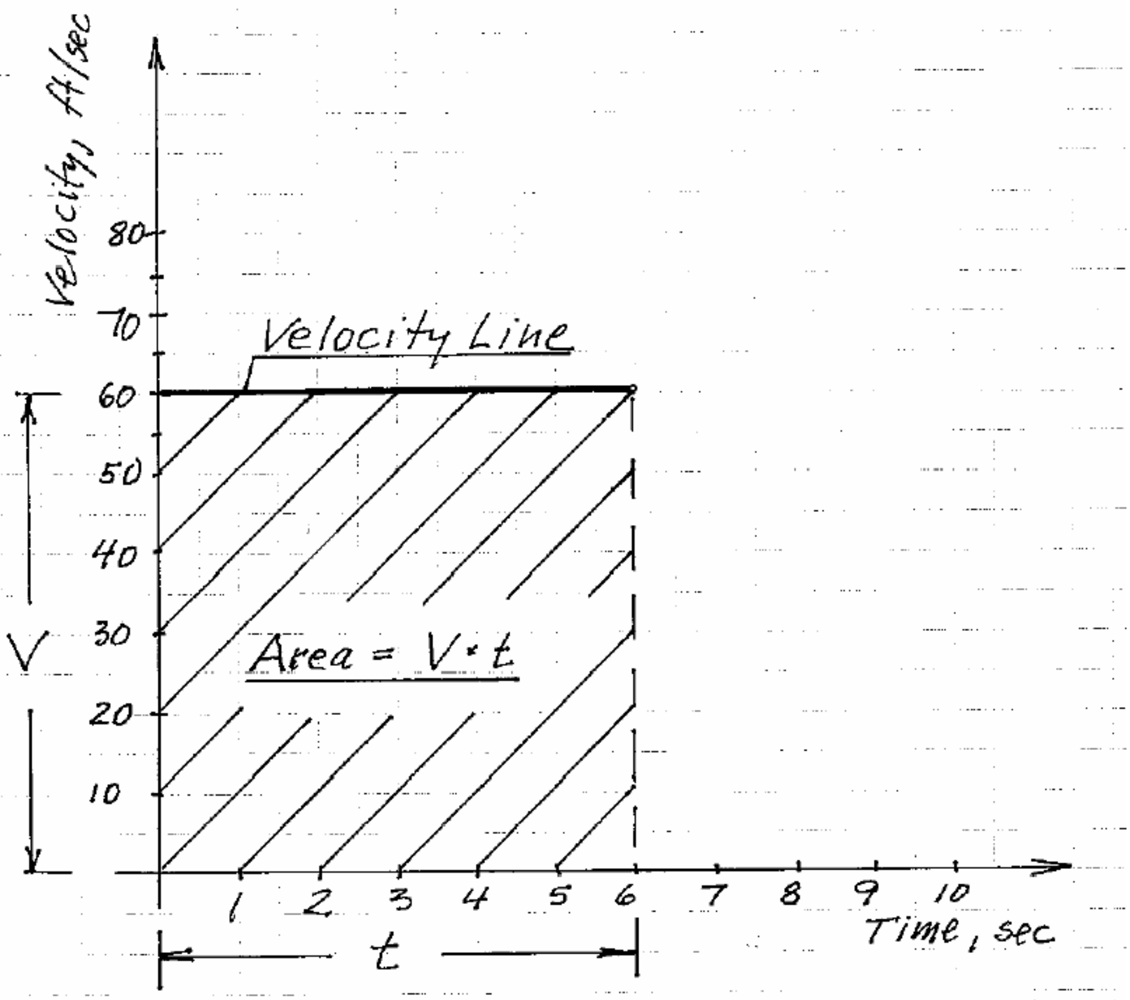

Figure 1 – Velocity-Time Diagram
Constant Velocity

This velocity-time diagram solution approach to complex two-vehicle motions requires us to only remember the basic definition equation $a = V/t$. The distance equation is developed from the V-t Diagram.

3.0. APPLICATIONS

3.1. VEHICLE DECELERATING TO A STOP – CONSTANT DECELERATION

The velocity-time diagram is illustrated in Figure 2. The area under the velocity line is a triangle. The area of the triangle is calculated by

$$\text{Area} = \text{height x width}/2$$

With the symbols shown in Figure 2 the area, and hence, the distance traveled is

$$S = Vt/2;\ \text{ft} \qquad (3)$$

Using Step 4 above, namely

$$a = V/t = (V_2 - V_1)/(t_2 - t_1);\ \text{ft}/(\text{sec}^2) \qquad (4)$$

allows us to derive any motion equation for a vehicle coming to as top with constant deceleration a. For example, solving Equation 3 for time and substituting into Equation 4 yields the well known relationship

$$S = V^2/(2a);\ \text{ft} \qquad (5)$$

The number 2 in the denominator results from the triangle area equation. All motion equations for a vehicle decelerating to a stop are presented in Section 20-1(d) of Reference 1.

Equation 5 is the basic equation for computing speeds after impact or vehicles braking to a stop. Frequently, it is expressed in different units as:

$$S = V^2/(30f);\ \text{ft} \qquad (6)$$

where: V = velocity, mph
f = deceleration or drag factor, g-units

An “expert” once testified in trial in Salt Lake City that the 30 in Equation 6 simply resulted from 32.2 rounded down to 30. Wrong! The derivation of Equation 6 starting with Equation 5 is shown in Section 20-4(a) of Reference 1.

Derivation of motion equations requires a constant deceleration during the braking process. If the braking process involves deceleration increasing from zero to its maximum sustained level, then the velocity line, during the time period in which the

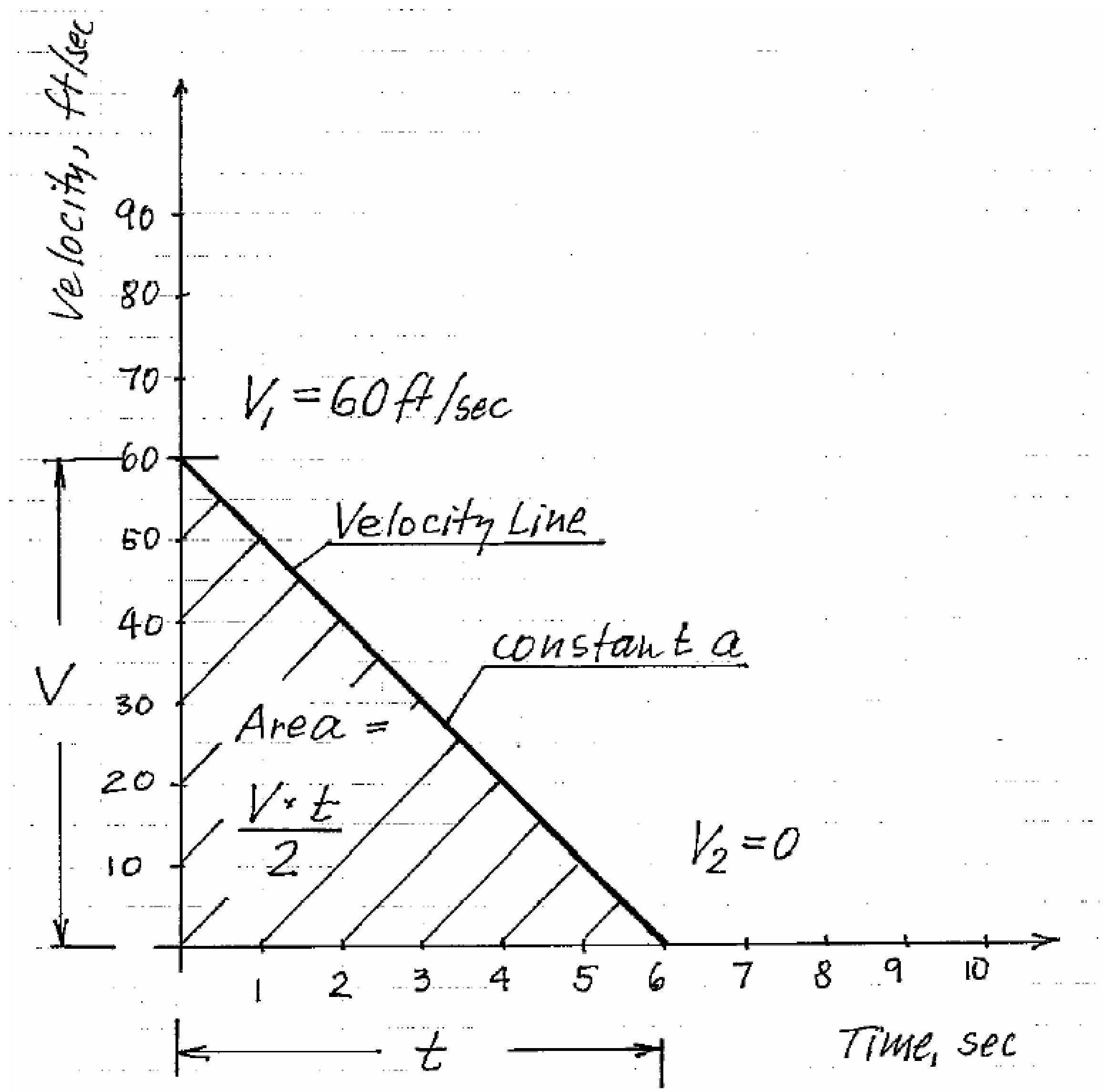

Figure 2 - Vehicle Decelerates to Stop

deceleration increases, is a curved line. The entire stopping distance derivation using calculus for this braking process involving a linear increase of deceleration from zero to its maximum level is presented in Section 1.4.3, Expanded Stopping Distance Analysis, of Reference 3.

Equation 5 would be modified as:

$$S = V^2/(2a_{max}) - a_{max}(t_b)^2/(24);\ ft \qquad (7)$$

where: a_{max} = maximum sustained deceleration, ft/sec^2
t_b = deceleration build-up time, sec

For example, for a braking process involving skidding tires with a deceleration of 26 ft/sec^2 (0.81g) and a deceleration build-up time t_b = 0.6 sec, the negative term in Equation 7 is only 0.39 ft. Only in accidents where the driver uses significant time greater than 1 sec to reach maximum deceleration (and low speeds) may the negative term become important. For example, for V = 20 mph and t_b = 2 sec, S = 16.53 – 4.33 = 12.2 ft, and not 16.53 ft.

3.2. DECELARATING FOM INITIAL TO FINAL VELOCITY

Consider a car traveling at V_1 = 60 mph (88 ft/sec), the driver applying the brakes, the vehicle decelerating at a = 8 ft/sec^2 (0.25g) to V_2 = 25 mph (37 ft/sec) when impacting a solid wall. The velocity-time diagram is shown in Figure 3.

The area under the velocity line is a trapezoid, consisting of a rectangle and a triangle. The area of the rectangle is equal to V_2 t, the area of the triangle equal to $(V_1 – V_2)$ t /2. Consequently, the distance S traveled by the car while braking is equal to:

$$S = V_2 t + (V_1 - V_2)\ t/2;\ ft \qquad (8a)$$

$$S = (V_1 + V_2)\ t/2;\ ft \qquad (8b)$$

Equation 8b can also be derived directly from the area of a trapezoid.

The area of the triangle in Figure 3 can also be expressed with $V_1 – V_2$ = at, resulting in:

$$S = (V_1)(t) - (at)(t)/2 = V_1 t - at^2/2;\ ft \qquad (9a)$$

or:

$$S = V_2 t + at^2/2;\ ft \qquad (9b)$$

With a = $(V_1 – V_2)$/t, all other equations shown in Section 20-1(f) of Reference 1 can be derived.

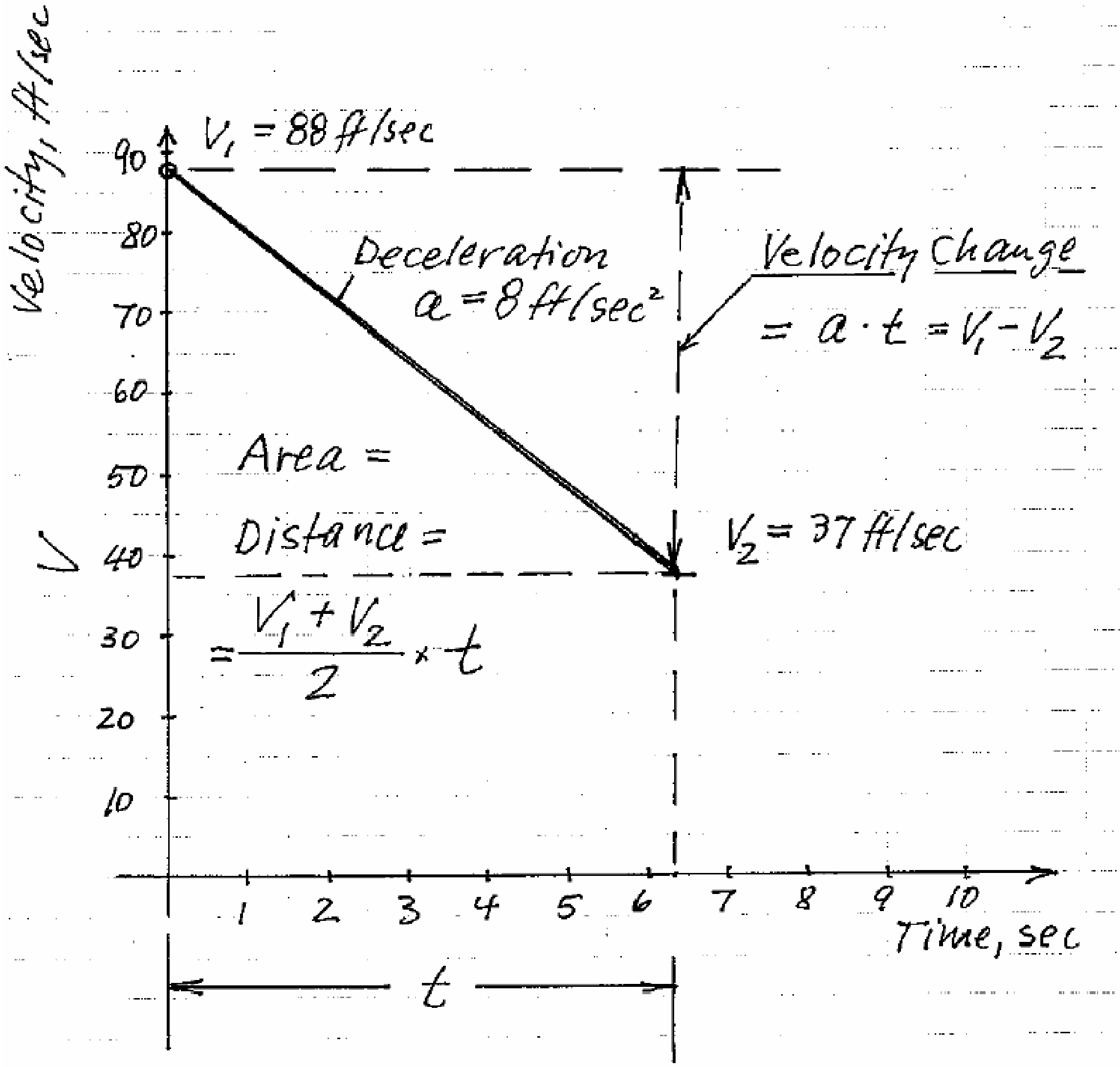

Figure 3 Decelerating Car with Final Velocity

3.3. AVERAGE VERSUS CONSTANT SPEEDS

A car travels from A to B at a constant speed of 100 ft/sec, and then returns from B to A again at a constant speed of 100 ft/sec. In a second trip the car travels from A to B at a constant speed of 140 ft/sec and then returns from B to A at a constant speed of 60 ft/sec. Formulate the motion problem for the two round trips.

The velocity-time diagram is shown in Figure 4. Since the distances from A to B are the same for car 1 and 2, the travel of car 2 is:

$$140\ t_{2ab} = 100\ t_{1ab}\ ;$$

or $$t_{2ab} = 0.714\ t_{1ab};\ \text{sec}$$

$$60\ t_{2ba} = 100\ t_{1ba};$$

or $$t_{2ba} = 1.667\ t_{1ba};\ \text{sec}$$

The travel time for car 1 is $2t_{ab}$, for car 2 is $(0.714 + 1.667)\ t_{1ab} = 2.38\ t_{1ab}$. Consequently, Car 2 requires 2.38/2 = 1.19 or 19% more travel time for the round trip. For example, for a distance of A to B of 1000 ft, car needs 2000/100 = 20 sec, car 2 needs 1000/140 = 7.14 sec plus 1000/60 = 16.667 sec totaling 23.8 sec.

3.4. RELATIVE MOTION BETWEEN TWO VEHICLES

Two vehicles accelerate from zero. Car 1 accelerates at 5 ft/sec^2 and after 16 sec has pulled ahead of car 2 by 148 ft. What is the acceleration of car 2?

The velocity-time diagram is shown in Figure 5. The distance traveled by car 1 during 16 sec minus the distance traveled by car 2 equals 148 ft. Consequently, we have:

$$a_1\ t^2/2 - a_2\ t^2/2 = 148\ \text{ft}$$

For t = 16 sec and a_1 = 5 ft/sec^2, acceleration a_2 = 3.84 ft/sec^2.

3.5. BUS COMIMG TO A STOP

A bus approaches its bus stop 6500 ft away at a constant unknown speed. When closer to the stop, the driver slows the bus at a deceleration of 8 ft/sec^2. The total time to travel the 6500 ft is 120 sec including the braking time. Calculate the speed of the bus when it is 6500 ft from the bus stop.

The velocity-time diagram is shown in Figure 6. The total distance traveled is equal to a rectangle plus a triangle:

$$S = V_1t_1 + (V_1)^2/(2a_1) = 6500;\ \text{ft} \qquad (10)$$

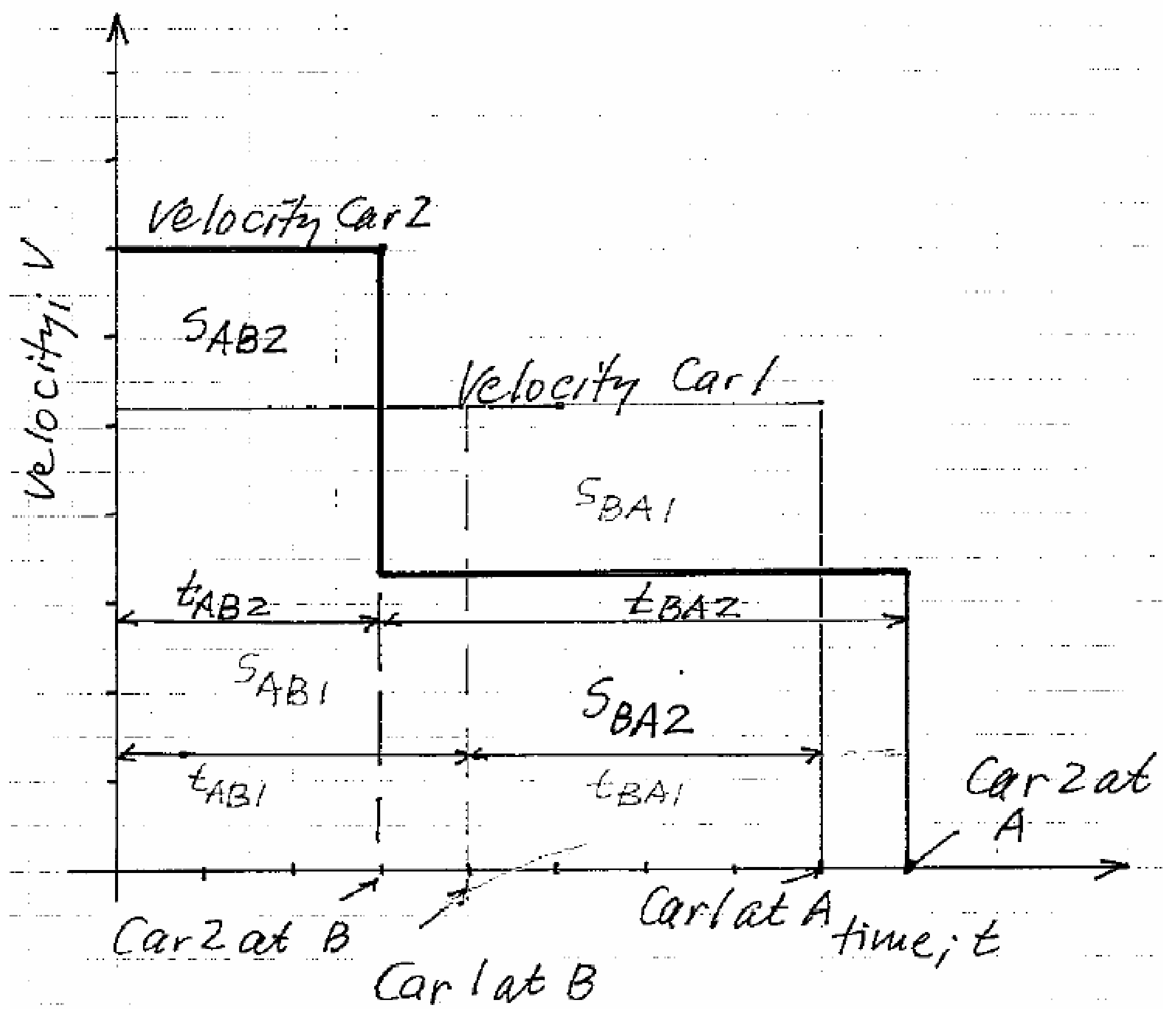

Figure 4 Average v. Constant Speeds

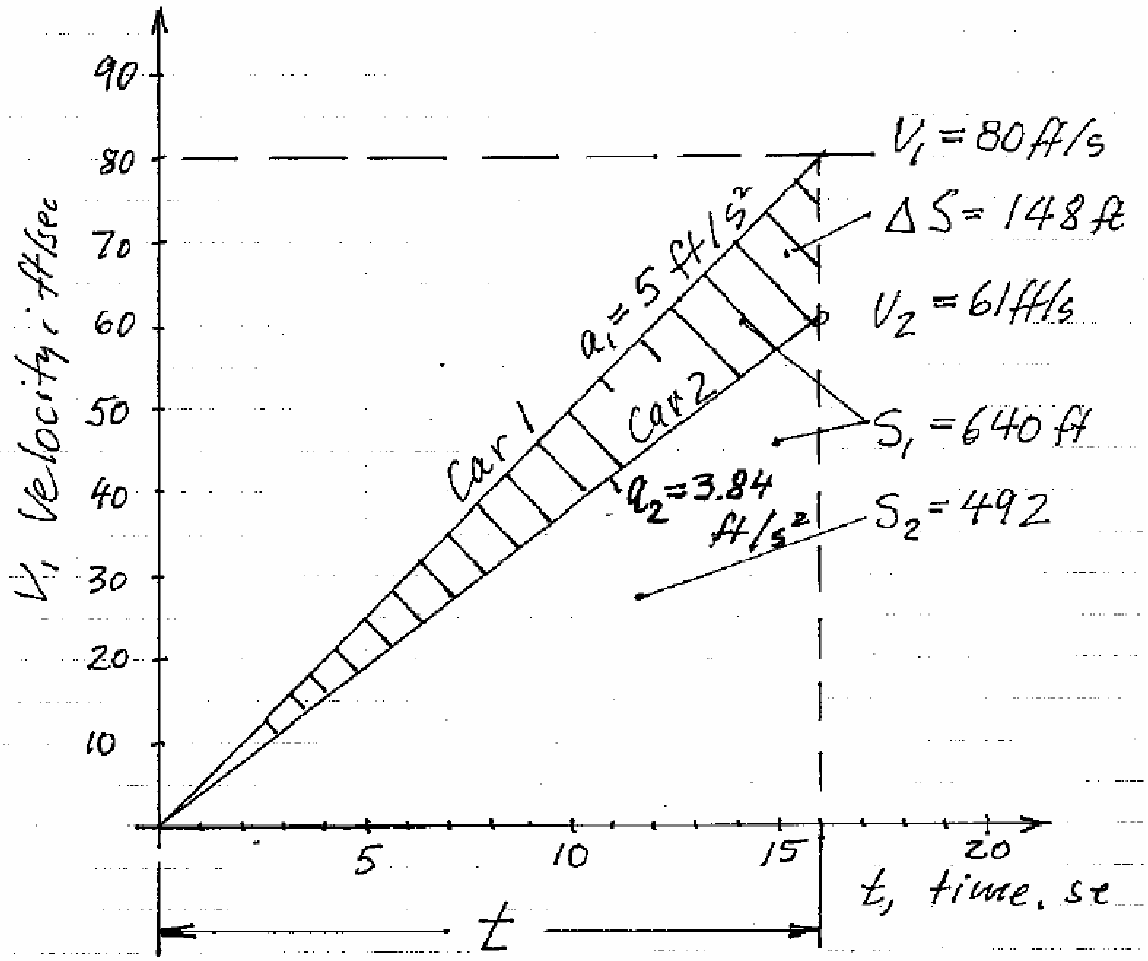

Figure 5 - Two Accelerating Vehicles

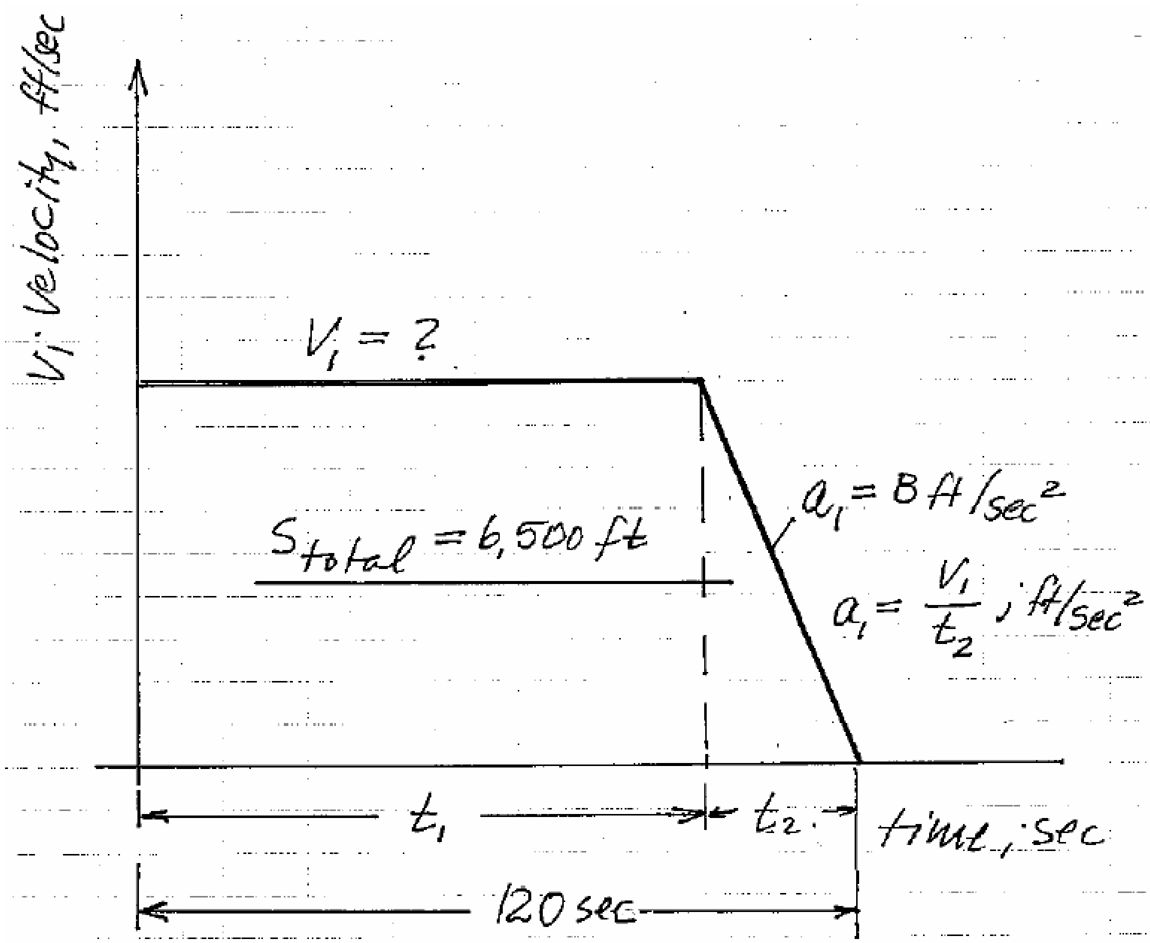

Figure 6 - Bus Coming to a Stop

The time equation is:

$$t_1 + V_1/a_1 = 120 \text{ sec} \qquad (11)$$

Solving Equation 11 for time t_1 and substituting into Equation 10, and solving for velocity of the bus at begin of travel yields:

$$V_1 = (120/2)\ a_1 – (((120/2)\ a_1)^2 – (6500)\ a_1)^{0.5};\ \text{ft/sec} \qquad (12)$$

Substituting $a_1 = 8$ ft/sec^2 into Equation 12 yields:

$$V_1 = 480 – (480^2 – (6500)(8))^{0.5} = 480 – 422.4 = 57.6 \text{ ft/sec}$$

or 39.3 mph.

3.6. MOTORCYCLE PASSES CAR

A car traveling at 25 mph passes a stationary motorcycle that at that moment begins to accelerate. The motorcycle passes the car after 30 sec. Calculate the acceleration and speed of the motorcycle at the moment of passing the car.

The velocity-time diagram is shown in Figure 7. The distance of the car and motorcycle are equal. Consequently, the areas of the rectangle and triangle are equal.

Hence,

$$V_c t = at^2/2;$$

or: $$a = 2\ V_c/t = (2)(36.65)/30 = 2.44 \text{ ft/sec}^2$$

and: $$V_{mc} = (2.44)(30) = 73.2 \text{ ft/sec (which is twice 25 mph)}$$

3.7. MAXIMUM SPEED TO STOP AT POINT OF IMPACT

In an accident the vehicle brakes before impact for a known distance. The total pre-impact distance from the point of driver reaction to the point of impact is called $S_{R-C.}$ Calculate the maximum speed the vehicle could have traveled and still come to a complete stop at the point of impact. We will derive Equation 30-2 of Reference 1.

The velocity-time diagrams for the actual and avoidance motion are shown in Figure 8. The areas, and hence distances, under the V_1- V_2 line, and the $V_{max} – V_2(0)$ line are the same.

Consequently, we have: $V_{max}t_R + (V_{max})^2/(2a) = S_{R-C}$

Solving the quadratic equation yields:

$$V_{max} = ((at_R)^2 + 2aS_{R-C})^{0.5} – at_{R;}\ \text{ft/sec} \qquad (13)$$

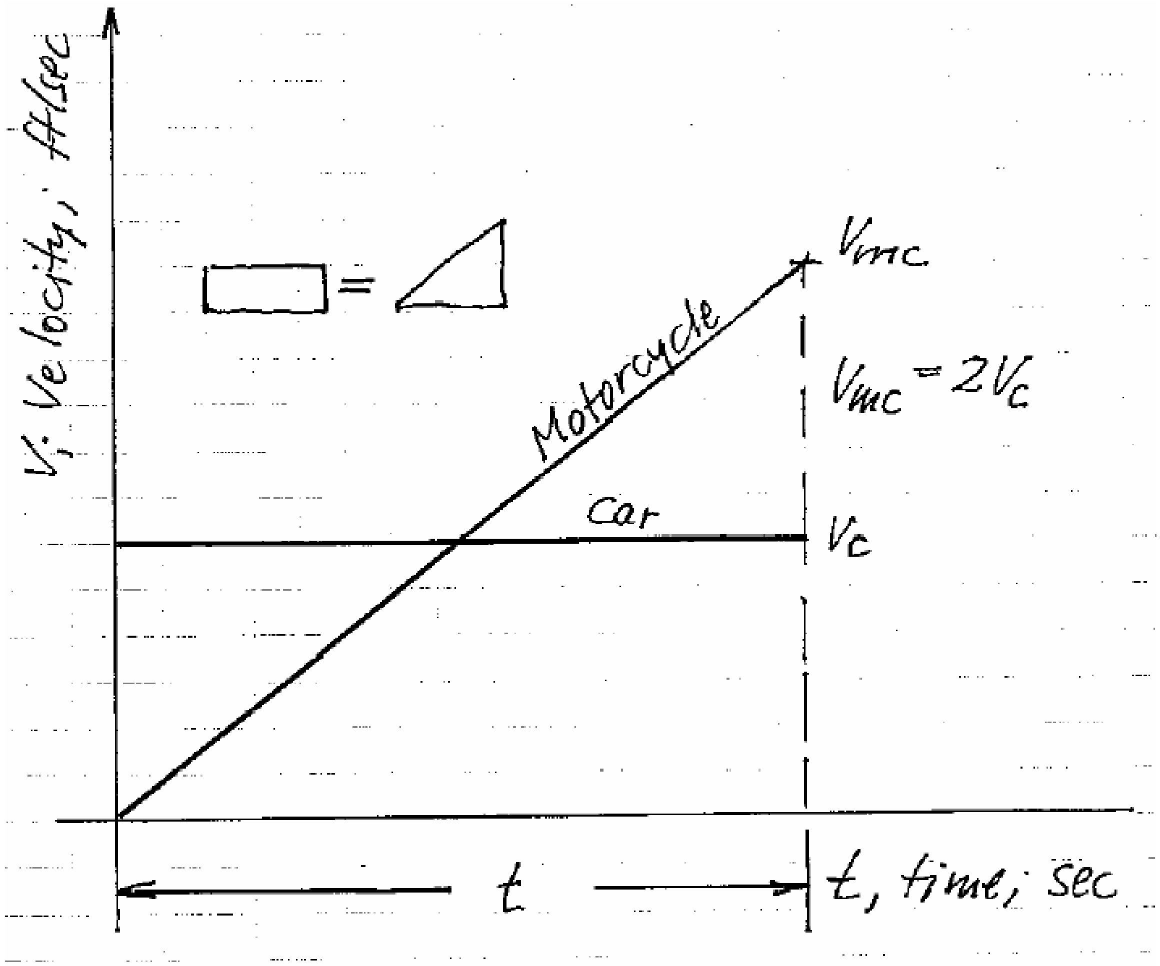

Figure 7 - Motorcycle Passes Car

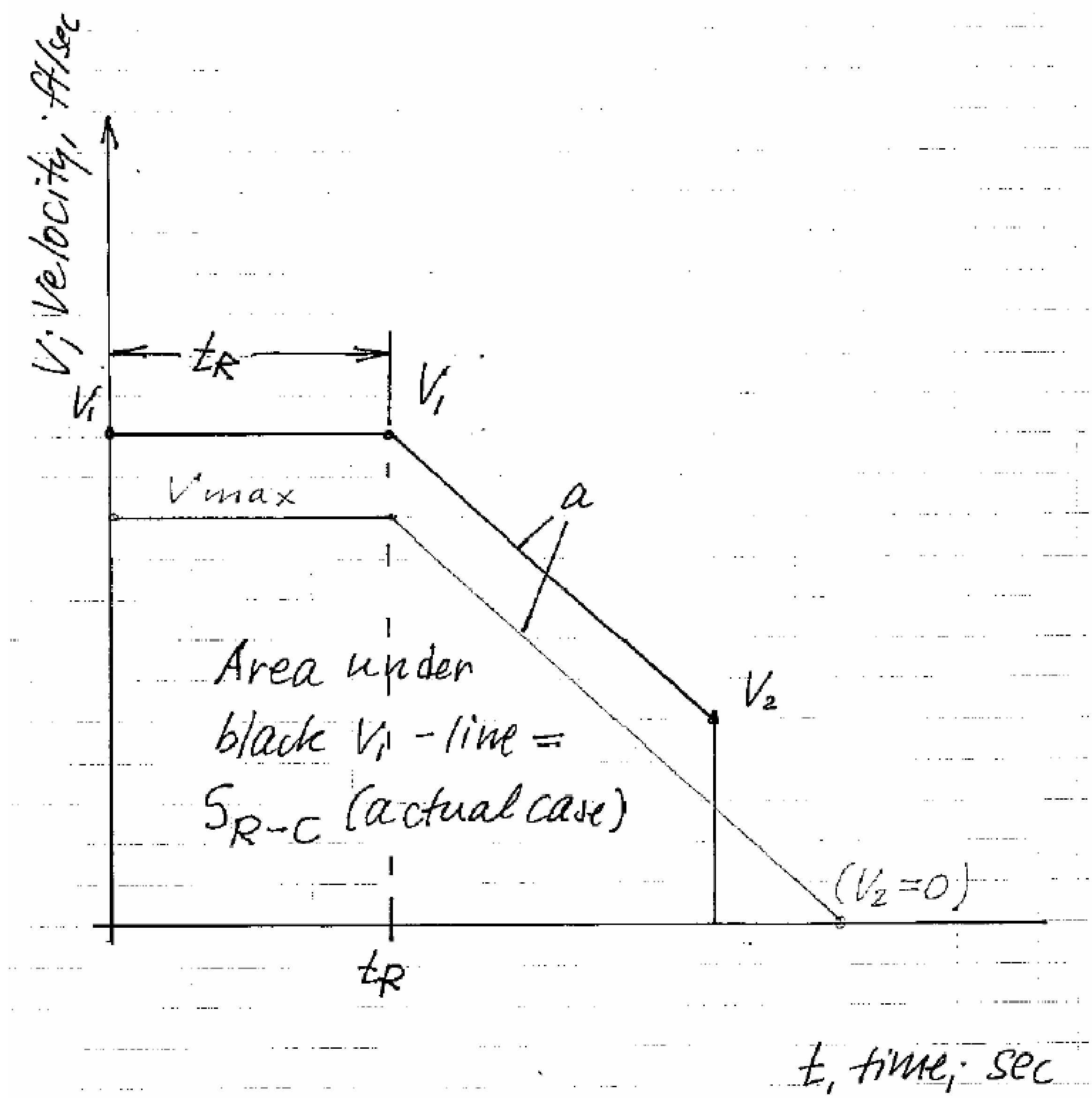

Figure 8 – Stopping at POI

$$V_{max} = ((at_R)^2 + 2aS_{R-C})^{0.5} - at_{R;}\ ft/sec \qquad (13)$$

Equation 13 is Equation 30-2 of Reference 1.

3.8. MAXIMUM SPEED FOR OTHER VEHICLE TO CLEAR IMPACT AREA

The accident is the same as in Section 3.7, except now we want the impacting vehicle to arrive at the point of impact sufficiently late for the other vehicle to have cleared the area of impact. We will derive Equation 30-4 of Reference 1.

The velocity-time diagrams for the actual and avoidance motion are shown in Figure 9. The area under the V_1- V_2 line is the distance traveled by the actual accident vehicle from driver reaction begin to impact, and is called S_{R-C}. The avoidance area under the V_{max}-V_3 line also equals S_{R-C}. The avoidance vehicle arrives the avoidance time t_{AV} later at the point of impact traveling at V_3. The avoidance time is determined by the speed and distance required by the struck vehicle to move out of the impact area. The time t_{B-C} is the time during which the actual vehicle was braking prior to impact.

The trapezoidal avoidance area or distance S_{R-C} equals the rectangle area minus the triangle area, resulting in

$$V_{max}(t_{R-C} + t_{AV}) - (a/2)(t_{B-C} + t_{AV})^2 = S_{R-C}$$

Solving for the maximum avoidance speed yields:

$$V_{max} = ((a/2)(t_{B-C} + t_{AV})^2 + S_{R-C})/(t_{R-C} + t_{AV});\ ft/sec \qquad (14)$$

3.9. SEMI REARENDS CAR

A semi travels at V_2 = 70 ft/sec (48 mph) behind a car traveling at V_1 = 108 ft/sec (74 mph). The driver of the car applies its brakes producing a deceleration of 0.8g, the semi while braking at 0.45g rear-ends the car. The crush damage sustained by the car (and front end of the truck) indicates a relative speed at impact of 28 ft/sec (19 mph) (See Equation 33-14 of Reference 1). Calculate the time period required by the driver before his brakes were locked, or in other words, determine if the truck driver reacted to the stopping car in a reasonable time?

The combined velocity-time diagrams are illustrated in Figure 10. We know that the difference in velocity at impact has to be 28 ft/sec as obtained from the crush damage sustained by the car (and tractor).

Close inspection of the rear damage sustained by the car revealed that the rear wheels were rotating at the moment of impact most likely indicating that the car was moving when impacted.

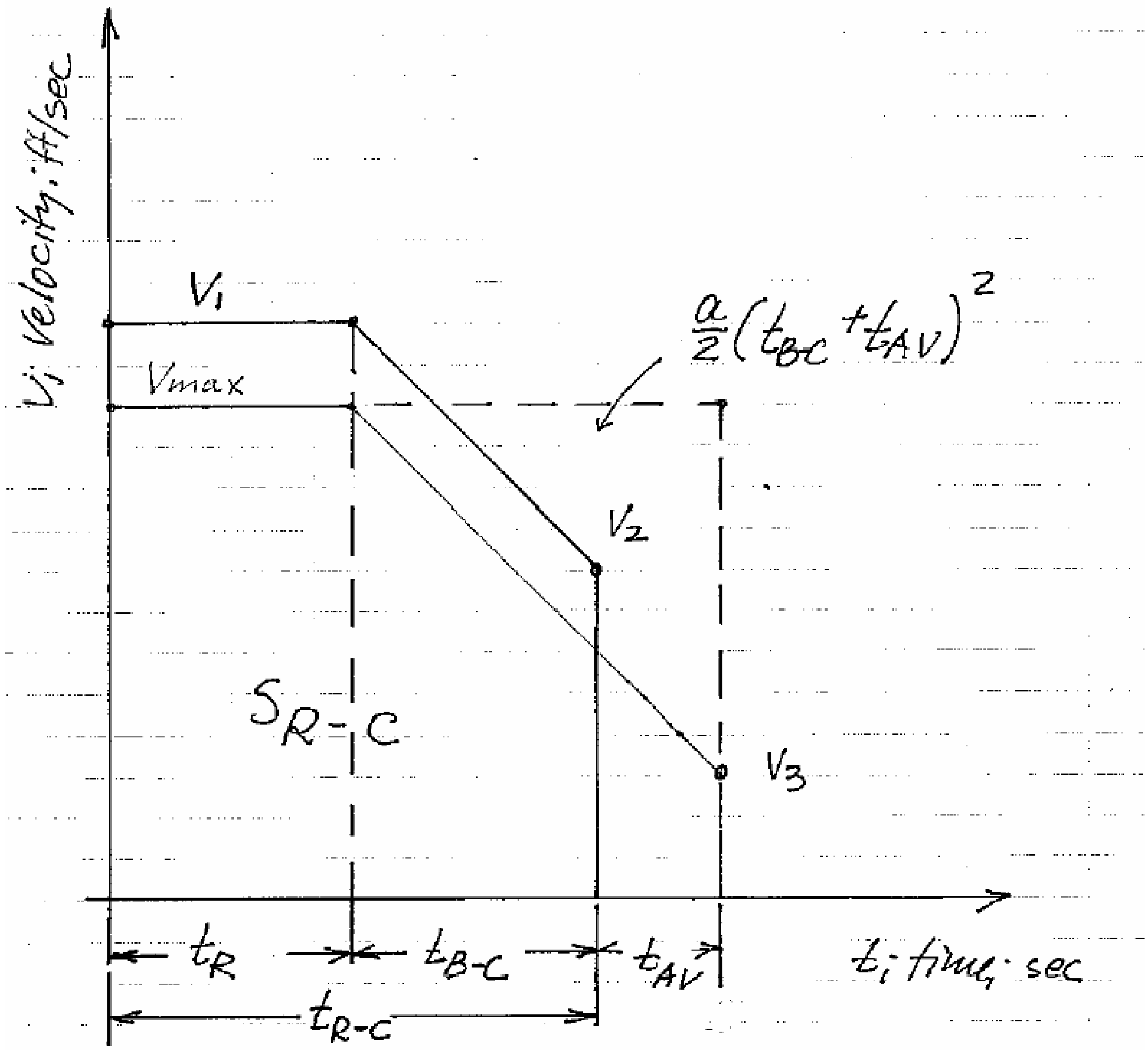

Figure 9 - Crash Avoidance by Delaying Arrival by t_{AV}

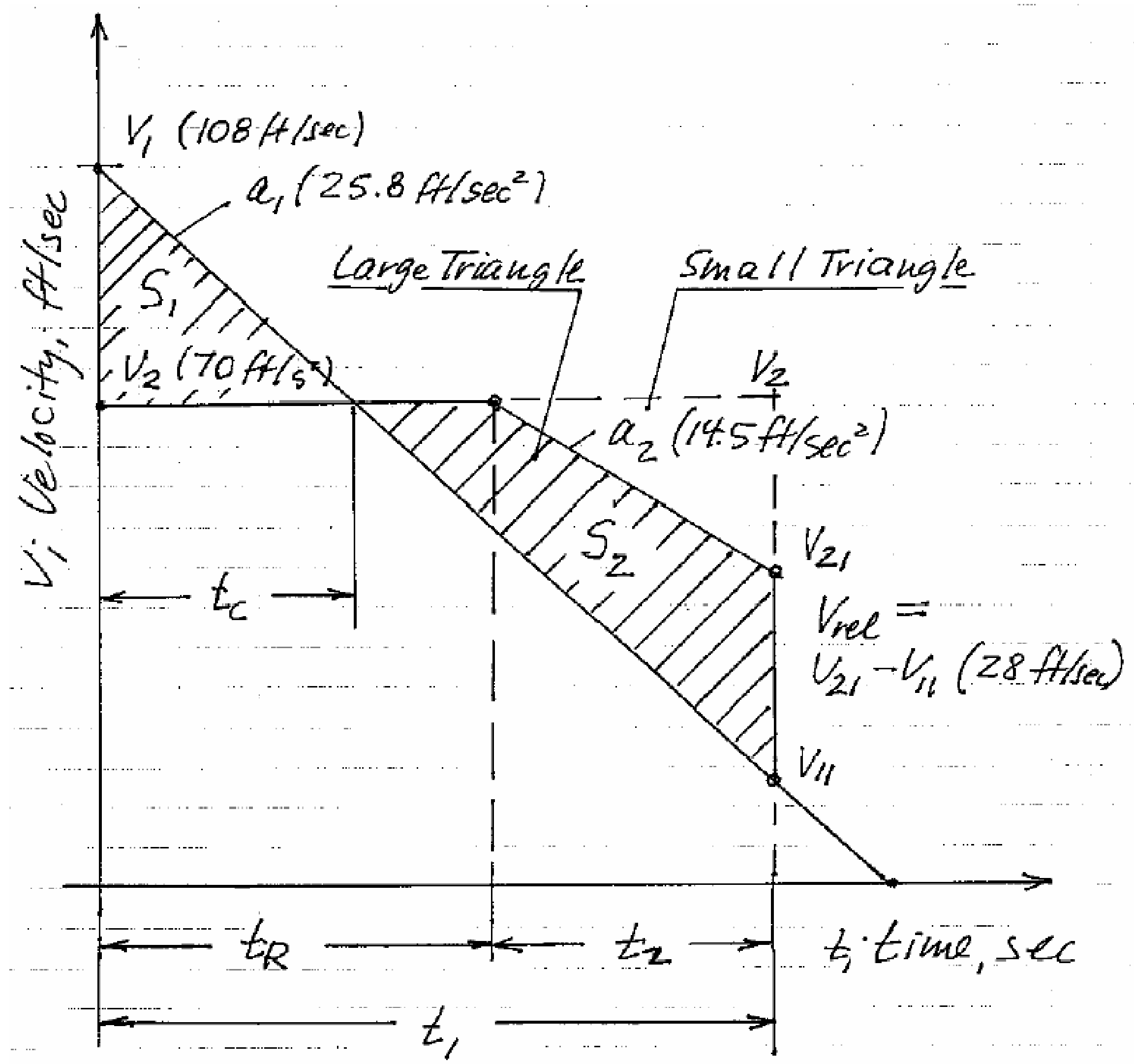

Figure 10 - Semi Rearends Car

Since the areas under the respective velocity lines V_1-V_{11} (car) and $V_2 – V_{21}$ (truck) have to be equal, we know that area S_1 (small triangle) has to equal area S_2 (small four-corner area).

Area S_1 can easily be calculated as:

$$S_1 = (V_1 – V_2)^2/(2a_1) = (108 – 70)^2/(2x25.8) = 28\ ft^2$$

Area S_2 can easily be calculated by subtracting a small triangle from a larger one:

$$S_2 = (1/2)(V_2 – V_{11})(t_1 – t_c) – (1/2)(V_2 – V_{11} – V_{rel})t_2 = S_1 \quad (15)$$

The time for the car to slow from V_1 to V_2 can easily be calculated from:

$$t_c = (V_1 – V_2)/a_1 = (108 – 70)/(25.8) = 1.47\ sec$$

The other equations follow from step 4 as:

$$V_2 – (V_{11} + V_{rel}) = a_2t_2 \quad (16a)$$

and:

$$(V_1 – V_{11}) = a_1t_1 \quad (16b)$$

Finally, the reaction time of the driver is $t_R = t_1 – t_2$; sec

Equation 15 has three unknowns, which can be solved by using Equations 16. The algebra is lengthy, resulting in the travel speed V_{11} of the car at impact:

$$V_{11} = - A/2 + ((A^2) + B)^{0.5};\ ft/sec \quad (17)$$

where: $A = (t_ca_1 – V_1 – V_2 + 2(a_1/a_2)(V_2 – V_{rel}))/(1 – a_1/a_2);\ ft/sec$ (17a)

$$B = (V_1V_2 – t_ca_1V_2 – (a_1/a_2)((V_2)^2 – 2V_2V_{rel} + (V_{rel})^2)/(1 – a_1/a_2); \quad (ft/sec)^2 \quad (17b)$$

The analysis for the particular accident avoidance analysis yields the following results:

V_{11} = 28.5 ft/sec (19.4 mph); V_{21} = 56.1 ft/sec (38.5 mph); t_1 = 3.08 sec; t_2 = 0.93 sec; t_R = 2.15 sec. An accuracy check can be done by computing the trapezoidal area with the calculated parameters to see if it equals 28 ft^2. Would the accident have been avoided, if the driver had reacted within 1.25 sec? What would have happened if the tractor brakes had been out-of adjustement?

3.10. ACCELERATING CAR IS REAR-ENDED

Vehicle 2 pulls out of a driveway and accelerates into highway traffic when vehicle 1 is S_{1-2} feet behind vehicle 2 at a speed of V_1. Vehicle 1 brakes but rear-ends vehicle 2. The relative velocity V_{rel} impact is known from crush damage. If the acceleration of vehicle 2 and deceleration of vehicle 1 as well as the speed of vehicle 1 are known, calculate the driver reaction time of vehicle 1.

The problem formulation is similar to Section 3.9. The velocity-time diagram is illustrated in Figure 11. The area under the V_1-line minus the area under the V_2-line has to equal the distance $S_{1\text{-}2}$ at the beginning of the motion process. Consequently, we have:

$$V_1 t_2 - (V_1 - V_{11}) t_1/2 - V_{21} t_2/2 = S_{1\text{-}2};\ \text{ft} \qquad (18)$$

Step 4 equations are:

$$V_1 - V_{11} = a_1 t_1 ;\ \text{ft/sec} \qquad (19)$$

and:

$$V_{21} = a_2 t_2 ;\ \text{ft/sec} \qquad (20)$$

The last equation is: $V_{rel} = V_{11} - V_{21}$; ft/sec (21)

Equation 18 contains four unknowns, which can be solved by use of the other equations, for example for the speed V_{11} of vehicle 1 at impact as:

$$V_{11} = A/2 - ((A/2)^2 - B)^{0.5};\ \text{ft/sec} \qquad (22)$$

where: $A = 2(V_{rel} + V_1 + V_1 a_2/a_1)/(1 + a_2/a_1)$; ft/sec

$$B = (2V_1 V_{rel} + a_2/a_1 (V_1)^2 + (V_{rel})^2 + 2a_2 S_{1\text{-}2})/(1 + a_2/a_1);\ (\text{ft/sec})^2$$

The reader must realize, that the input data must be realistic. For example, $V_1 = 60$ ft/sec, $V_{rel} = 20$ ft/sec, $a_1 = 22$ ft/sec^2, $a_2 = 6$ ft/sec^2 and $S_{1\text{-}2} = 250$ ft yield $V_{11} = 55.9$ ft/sec. The braking time is only $t_2 = 0.186$ sec. The driver reaction time would be equal to 5.79 sec in order fro the 250 ft separation distance to be achieved in this problem.

3.11. TRACTOR-TRAILER RUNS OVER MOTORCYCLE

At nighttime, a motorcycle without tail light was rear ended by tractor-trailer. The point of impact and the dual tire skid marks are shown in Figure 12. Only skid marks from the trailer tires were produced. The electronic data recorder DDEC of the truck showed a travel speed of 67mph as shown in Figure 13.

We have to determine what the braking effectiveness of the tractor-trailer was, and if and under what circumstances the impact could have been avoided by the operator of the truck.

The velocity-time trace produced by the DDEC is analyzed in Figure 14 to determine what decelerations the tractor-trailer actually produced. The velocity line slope between points 1 and 2 is equal to:

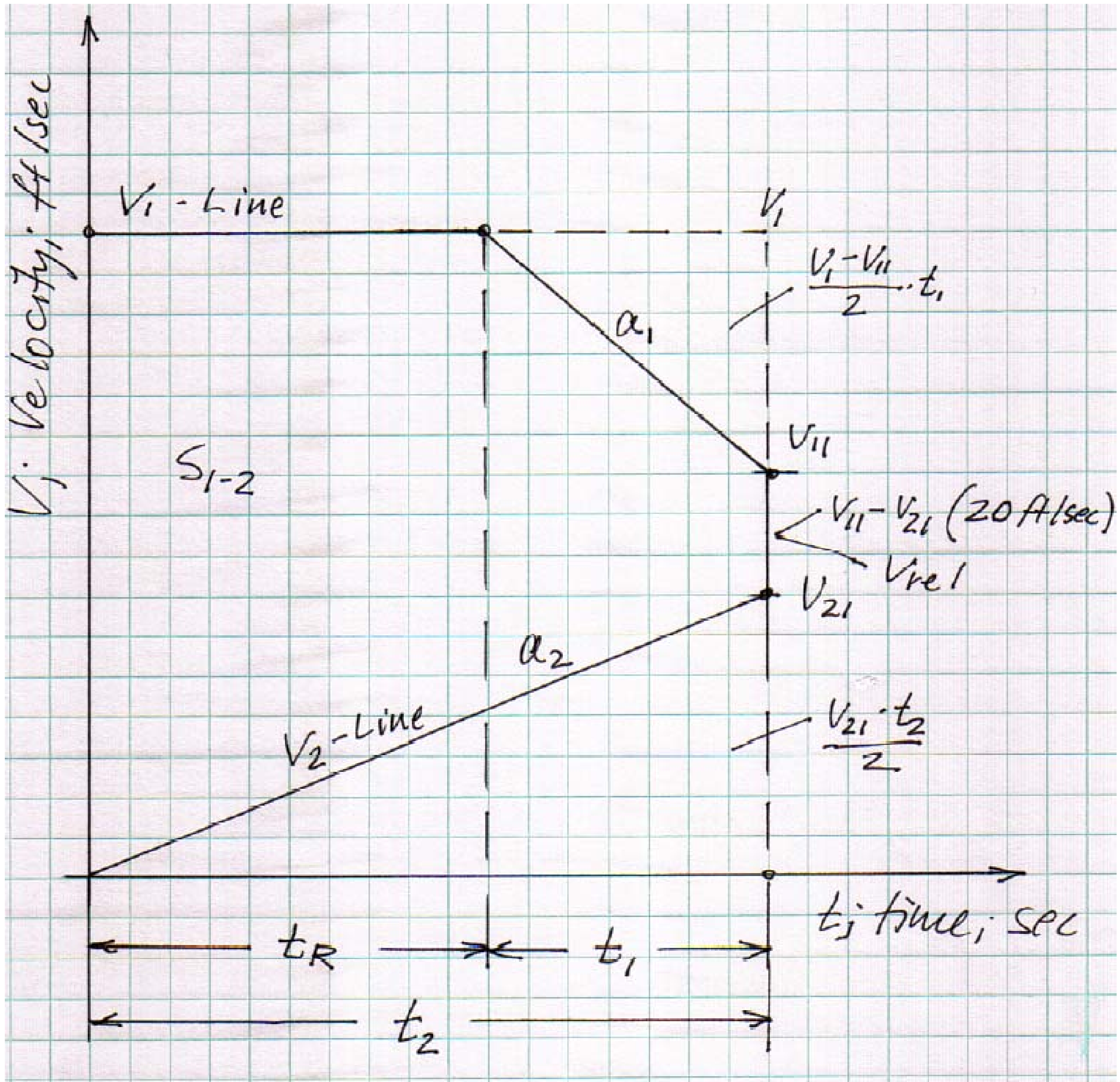

Figure 11 - Accelerating Vehicle Is Rear-Ended

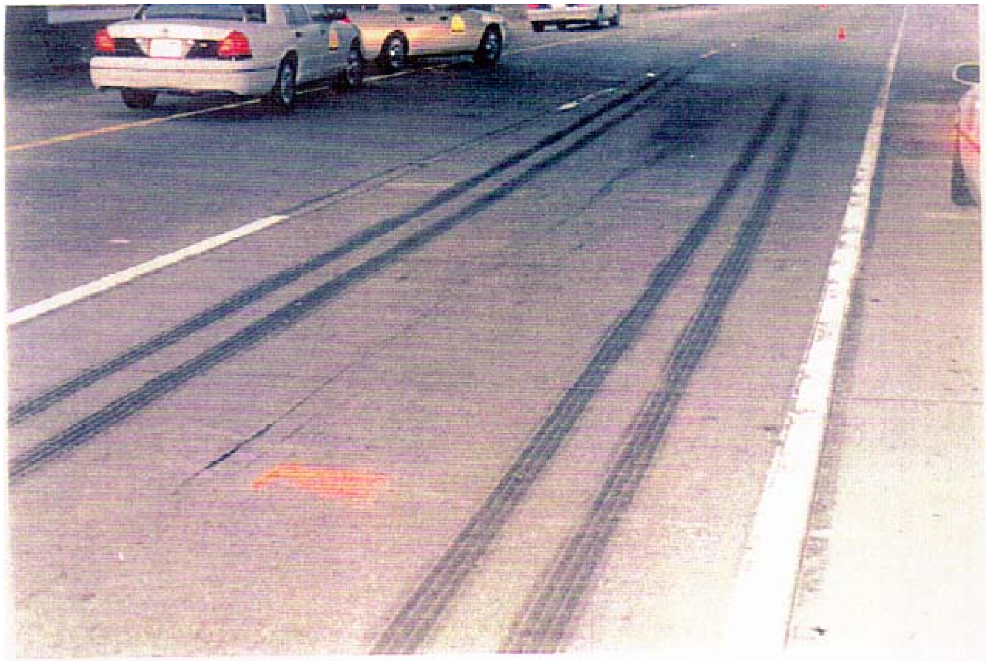

Figure 12 - Semi/Motorcycle POI

DDEC® Reports - Hard Brake #1

Print Date: Jul 17, 2001 01:29 PM (MDT)

Detroit Diesel-Allison

		Trip:	11/21/2000 to 07/17/2001 (EST
		Vehicle ID:	
		Driver ID:	
		Odometer:	62775.4 mi
Trip Distance	62775.4 mi	Trip Time	1585:07:43
Trip Fuel	10436.13 gal	Fuel Consumption	6.58 gal/h
Fuel Economy	6.02 mpg	Idle Time	493:33:53
Avg Drive Load	50 %	Idle Percent	31.14 %
Avg Vehicle Speed	57.5 mph	Idle Fuel	373.13 gal

Incident Time: 06/26/2001 05:44:56 (EST) Incident Odometer: 62775.2 mi

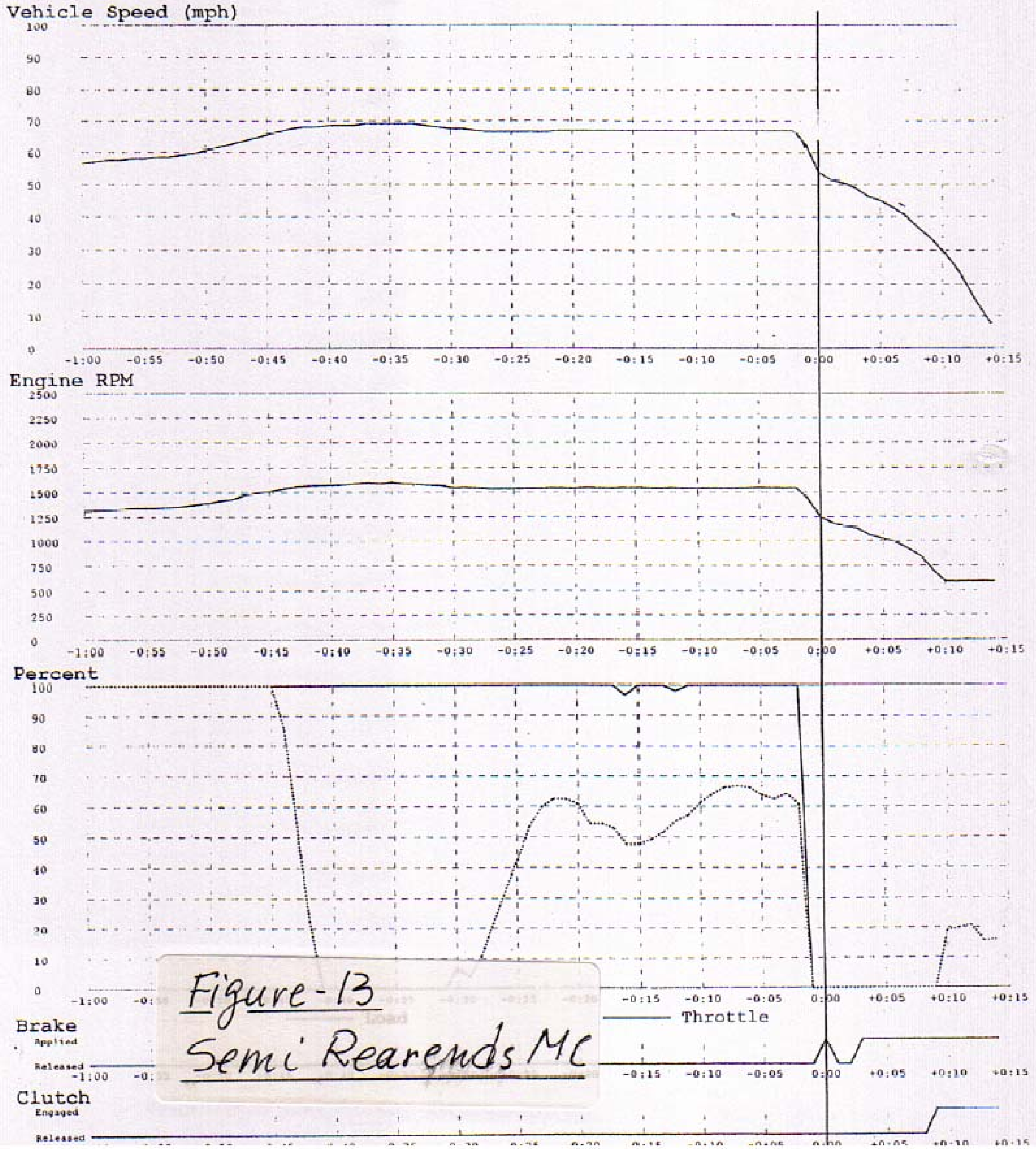

Figure-13
Semi Rearends MC

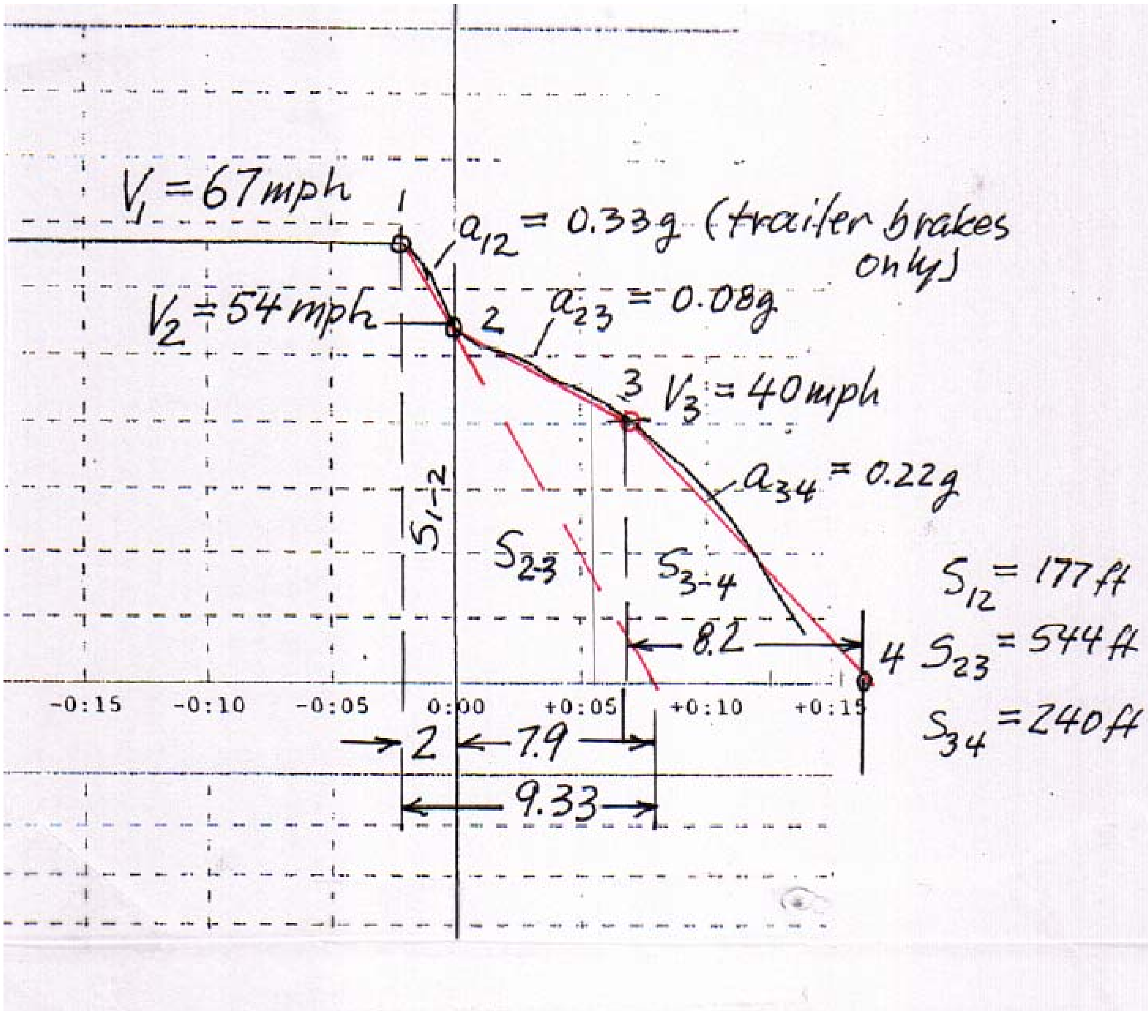

Figure 14 - Engine Data Analysis

$$a_{1\text{-}2} = (67)(1.466)/9.33 = 10.52 \text{ ft/sec}^2 \text{ or } 0.33g$$

The 9.33 sec is the time required by the truck if it had come to a complete stop with a 0.33g deceleration.

The deceleration between points 2 and 3 is:

$$a_{2\text{-}3} = (54 - 40)(1.466)/(7.9) = 2.6 \text{ ft/sec}^2 = 0.08g$$

The deceleration between points 3 and 4 is:

$$a_{3\text{-}4} = (40)(1.466)/(8.2) = 7.15 \text{ ft/sec}^2 = 0.22g$$

Inspection of the decelerations indicates that the truck never produced decelerations consistent with dry freeway pavement of approximately 0.5g. The driver stated that he initially only applied the trailer brakes most likely consistent with trailer tire skid marks at the accident scene and a combination deceleration of 0.33g. The total distance traveled during the three different deceleration phases can be computed from the velocity lines as:

$$S_{1\text{-}2} = (67 + 54)(1.466)(2)/(2) = 177 \text{ ft}$$

$$S_{2\text{-}3} = (54 + 40)(1.466)(7.9)/(2) = 544 \text{ ft}$$

$$S_{3\text{-}4} = (40)(1.466)(8.2) = 240 \text{ ft}$$

The total distance from brake apply at 67 mph until the tractor-trailer stopped was approximately $S_{total} = 961$ ft. At a deceleration of 0.5g the stopping distance would only have been 300 ft. One obvious conclusion is that the tractor operator did not use his brakes effectively in stopping/slowing his vehicle.

For the accident voidance analysis we assume a tractor-trailer deceleration of 0.5g and a head light illumination distance of 150 ft. The specific question to be answered is: With the truck traveling at 67 mph, and a visibility of 150 ft, what is the minimum speed of the motorcycle so that the braking truck slows to the speed of the motorcycle when it arrive at the motorcycle.

The velocity-time diagrams of both the truck and motorcycle are illustrated in Figure 15. The diagram reflects the truck velocity line from begin of driver reaction until it has slowed the velocity of the motorcycle. The velocity line of the motorcycle is a straight line at an unknown velocity.

The area under the truck velocity line is the truck's distance S_1 from reaction begin until it has reached the velocity of the motorcycle. The area under the velocity line of the motorcycle is its distance S_2 from begin of truck operator reaction until the truck is immediately behind the motorcycle now also traveling at the speed of the motorcycle. The separation distance equation is:

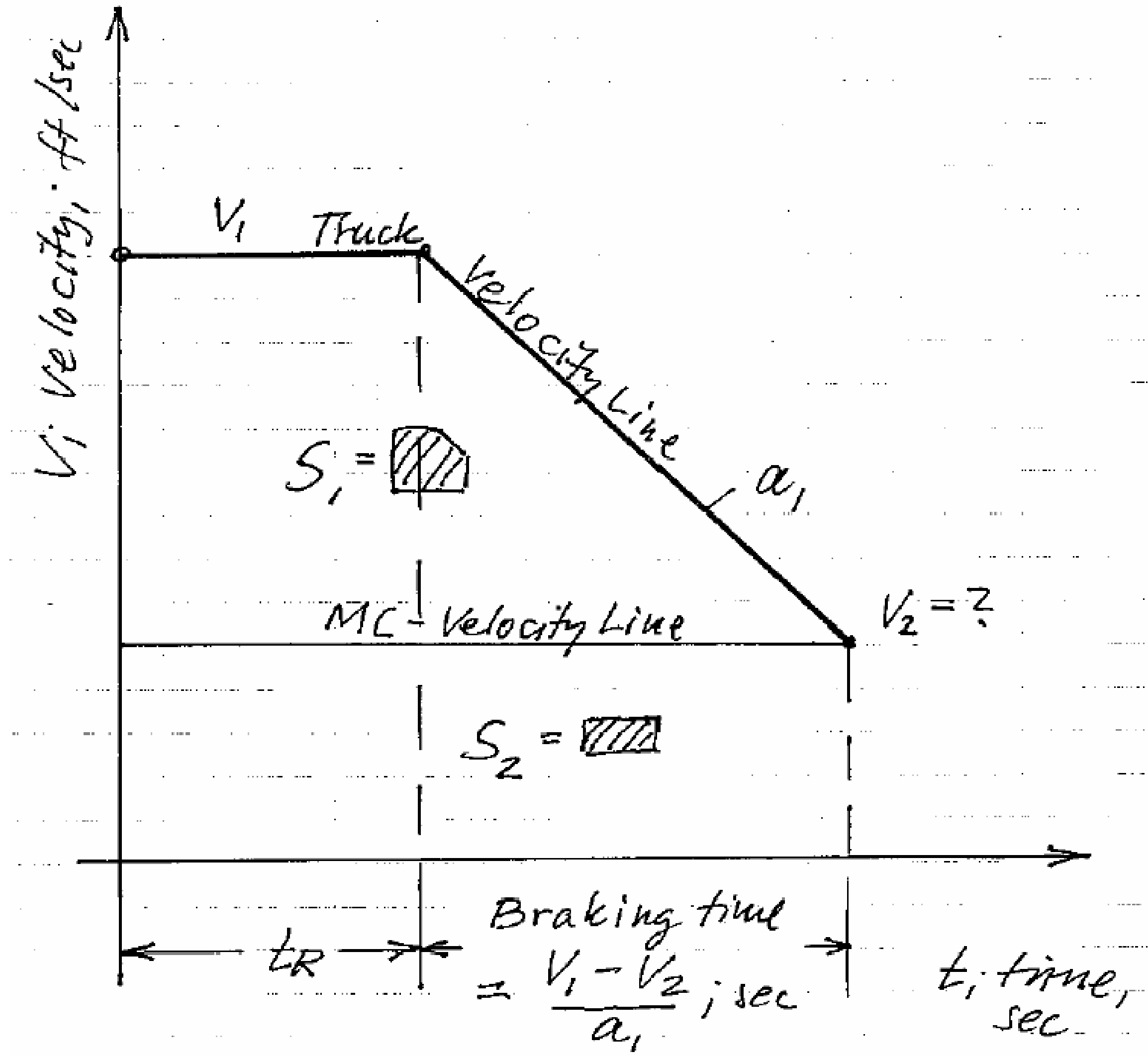

Figure 15 - Semi Rearends MC

$$V_1t_R + ((V_1)^2 - (V_2)^2/(2a_1) - (V_2t_R + (V_2)(V_1 - V_2)/(a_1) = S_1 - S_2;\ ft \qquad (23)$$

Solving Equation 23 for the speed of the motorcycle yields:

$$V_2 = A - (A^2 - B)^{0.5};\ ft/sec \qquad (24)$$

where: $A = V_1 + a_1t_R$; ft/sec

$B = (V_1)^2 + 2a_1t_RV_1 - 2a_1S_{1\text{-}2}$; $(ft/sec)^2$

Substitution of $a_1 = 16$ ft/sec^2, $V_1 = 98$ ft/sec, $t_R = 1.5$ sec and $S_{1\text{-}2} = 150$ ft, yields a motorcycle speed of $V_2 = 48.8$ ft/sec or 33 mph.

Consequently, we can conclude that if the motorcycle were traveling at 33 mph, and all the other factors as assumed, the crash would not have occurred. The reader is advised to develop "Can-Stop" chart as illustrated in Figure 16 by using different driver reaction times (night time accident usually do not involve 1.5 sec "normal" driver reaction time) and the view distances may actually be longer than only 150 ft. In Figure 16 the driver reaction time is $t_R = 1.5$ sec while the view distance was varied. In the actual case witnesses had stated that the motorcycle speed ranged between 25 and 30 mph.

4.0. CONCLUSIONS

The velocity-time diagram method changes a complex motion problem into a simple geometry problem, involving the calculation of areas.

The author has used the velocity-time diagram frequently to clearly formulate a solution plan to complex motion problems. When required, the basic steps were explained to juries who needed to understand the fundamentals involved, however, without being dragged into the algebraic solutions of quadratic equations. Many examples used in this paper were derived from actual cases used by the author in trial.

Some times the solution process can become fairly involved algebraically, however the reader is advised to always involve Step 4 in the analysis. Memorizing certain formulas is not necessary since they can quickly be derived from simple geometric shapes.

References

1. Motor Vehicle Accident Reconstruction and Cause Analysis by Rudolf Limpert, Lexis-Nexis, Matthew Bender, 5th edition, 1999.

2. Mechanik und Festigkeitslehre, Alfred Boege, Vieweg Publisher (Germany), 1974.
3. Brake Design and Safety, by Rudolf Limpert, SAE International, 2nd Edition, 1999.

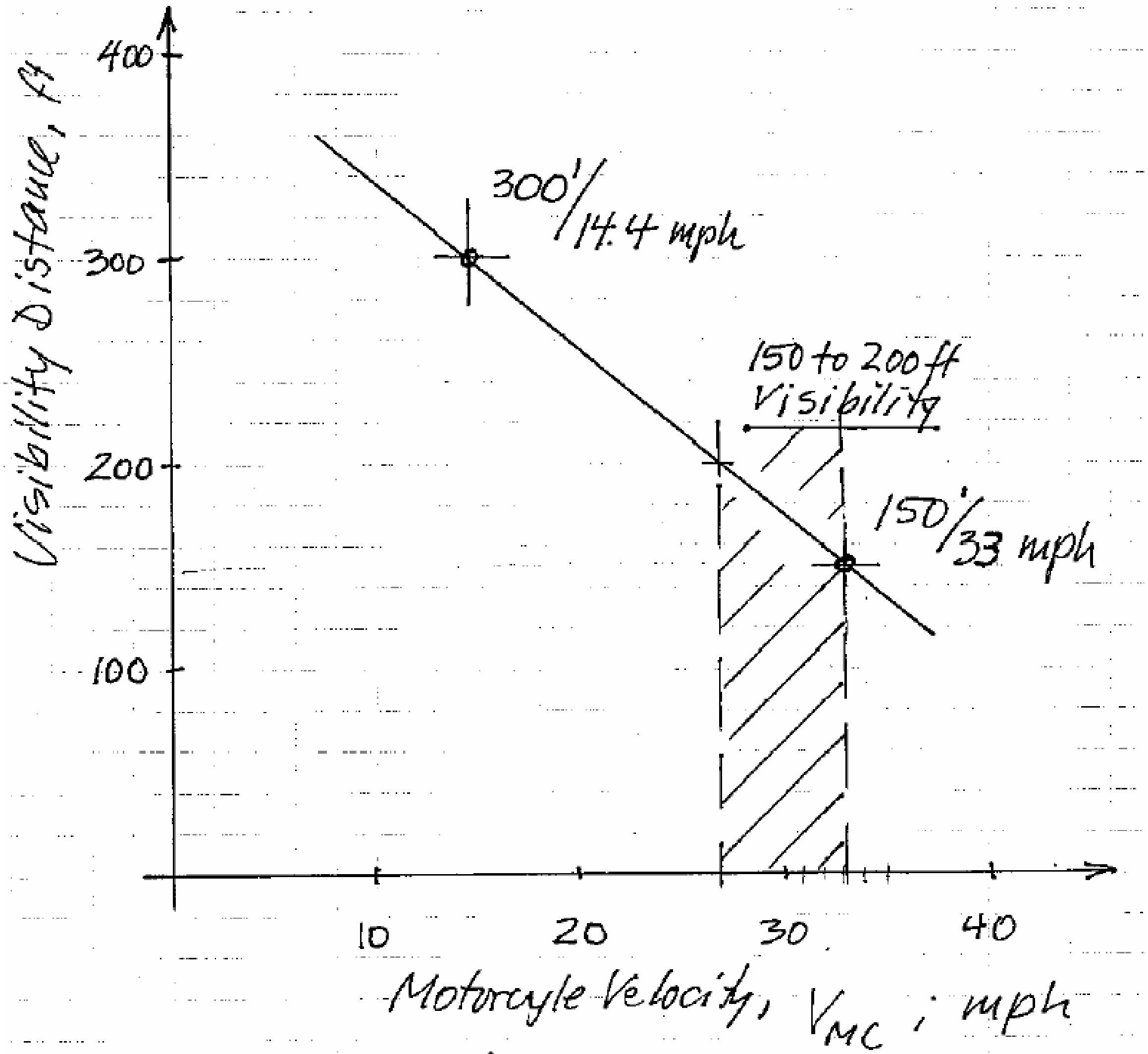

Figure 16 - "Can - Stop" Chart For Different Visibility ; $t_R = 1.5$ sec

References 1 and 3 can be ordered from the publishers by visiting our website and clicking on “Publications”.

Practice Problems

1. What are the initial velocity and acceleration of a car that travels 19.7 ft in the sixth second and 26.2 ft in the 11^{th} seconds? (12.5 ft/sec and 2.6 ft/sec^2).

2. Vehicle 1 travels at a constant speed of 37.2 mph behind a slower vehicle traveling at 26 mph. If the initial separation distance is 1312 ft, how much time and distance does the faster car need before it reaches the slower vehicle? (80 sec and 4373 ft)

3. A train traveling at 44.7 mph suffers a delay of 3 minutes because it temporarily must reduce its speed to 11.2 mph in a construction zone. Deceleration and acceleration are 0.66 and 0.33 ft/sec^2, respectively. How long is the construction zone? (2091 ft)

4. A driver sees a 30 mph speed limit sign when he is 164 ft away. For an initial speed of 50 mph, calculate braking distance and deceleration.(2.8 sec, 9.9 ft/sec^2)

5. Traffic on a freeway had come to a complete stop due to snowing conditions. A semi had stopped several feet behind a van. When the van began to move, the semi driver prepared to start moving, when he was rear-ended by another semi. Formulate the motion problem with the objective of estimating the minimum distance the semi had stopped behind the van. Assume equal weight for the semis, and icy road conditions. Which accident scene parameters must be available?

6. A bus travels at 25 mph in slow heavy traffic. The car in front of it is 30 ft away, also traveling at 25 mph. The car suddenly slams on the brakes decelerating at 0.85g. The bus runs into the back of the car at 10 mph. The bus driver applies his brakes within 1.2 seconds after the car’s brake lights came on. Comment on the mechanical condition of the bus brakes?

7. Two cars travel at 65 mph in the same lane. The lead car brakes suddenly at 0.8g. The second car does not brake and rear-ends the lead car. The relative speed at impact is 20 mph. What was the separation distance at the moment the lead car began braking?

8. A car accelerates from a stop sign and travels 70 ft between the third and fifth second. At the end of the fifth second the car side-impacts another car at 13 mph. What was the acceleration of the car and did it stop at the stop sign?