

SHORT PAPER PCB 10-2006

INLINE LOW IMPACT SPEED COLLISIONS

ENGINEERING EQUATIONS, INPUT DATA AND MARC 1 APPLICATIONS

By:

Dennis F. Andrews, Franco Gamero, Rudy Limpert

**PC-BRAKE, INC.
2006**

www.pcbraKEinc.com

PURPOSE OF PCB SHORT PAPERS

To provide the accident reconstruction practitioner with a concise discussion of the engineering equations and limiting factors involved, evaluation of critical input data, and the analysis of actual cases by use of the MARC 1 computer software.

Short Papers are available free of charge and can be obtained by visiting our website at

www.pcbrakeinc.com

We hope that our Short Papers will assist the practitioner in better understanding the limitations inherent in any derivation of engineering equations, to properly use critical input data, to more accurately and effectively formulate his or her case under consideration, to become a better prepared expert in the field of accident reconstruction, and to more effectively utilize the full potential of the MARC 1 computer program.

Comments and suggestions are always invited by visiting our Discussion Forum and/or by writing to:

PC-BRAKE, INC.
1071 E. Tollgate Road
Park City, Utah 84098

Throughout the Short Papers we will extensively reference the 5th Edition of “Motor Vehicle Accident Reconstructions and Cause Analysis” by Rudolf Limpert, the “Accident Reconstruction Catalog” (ARC) CD, as well as the MARC 1 software.

LOW IMPACT SPEED INLINE COLLISIONS

1.0 INTRODUCTION

Low impact speed collisions can be divided into two categories, namely those without braking and those with braking during the actual crash. For the case without braking the engineering fundamentals are simple. An accurate analysis or reconstruction is only made difficult by the selection of a proper coefficient of restitution. For the case with braking the mathematical analysis is significantly more complex.

The basic engineering equations and their derivations are presented in Chapter 43 in the Supplement to “Motor Vehicle Accident Reconstruction and Cause Analysis”, 5th edition, by Rudolf Limpert, published by Matthew Bender Co. of LEXIS NEXIS (The Text). We will only discuss certain derivation details as they help readers to understand proper use and limitations of MARC 1 Module W4, Low Speed Inline Collisions with Braking, and Module W2, Inline Collisions (without braking). The objectives of PCB 10 – 2006 are to provide accurate information and references, as well as general input data for a reliable low impact speed accident reconstruction.

In preparation for this paper we have reviewed information available on the internet. By simply searching under “Low Impact Speed”, a large number of articles were evaluated. The authors ranged from lawyers to insurance adjusters to individuals with advanced college degrees. An impact speed of 10 mph seems to be the magical number for an accident to classify as low speed crash. As general background information, they may be helpful to the accident reconstructionist. Not all statements are correct. For example, one PhD writes that Newton’s Third Law states that momentum is conserved. He probably meant Newton’s Second Law. On the other hand, we located an excellent publication (Reference 8). A reference list of excellent SAE publications and others dealing with low speed impact testing is provided at the end of this paper.

The second law does not state that momentum is conserved. It states that when a resultant or external force F is acting on a mass m , the mass will experience an acceleration a , expressed by Newton’s Second Law in the form of $F = ma$. This equation can be rewritten as $Ft = mV$, that is, impulse equals momentum, where V is velocity and t is time. If the external force F acts between times t_2 and t_1 , then the velocity changes between V_2 and V_1 . For example, a car, initially stationary, accelerating from a red light with a resultant drive thrust F over a specific time period causes the car to accelerate from zero to a higher velocity V_2 . See Section 21-6(b) of the Text for more detail.

During a collision between two automobiles, the contact force between both vehicles at the point of common velocity is acting during the very short collision time. The product of contact force and contact time is called impulse. The contact force derived from the impulse is very large. For example, if the impulse at the point of common velocity is 4,500 lbsec (head-on collision between two 3,300 lb cars at 30 mph each), and the crash time is 0.1 sec or 100 ms, then the average contact force is $4,500/0.1 = 45,000$ lb. On the other hand, when the impulse is small, as in low speed collisions, the contact force will

be smaller. For example, for the same head-on collision at 10 mph, the impulse will be 1,500 lbsec with a contact force of 15,000 lb. Under those conditions, resultant external forces such as braking or tire side forces cannot be excluded. For example, the 3,300 lb car with its brakes locked will produce a braking force of 3,300 lb for a tire/road friction coefficient of $f = 1$, or 22% of the contact force between both vehicles. Since we are talking about small numerical values of impact speed and delta-V in low impact speed cases, an incorrect formulation of the impact physics with braking will be unacceptable for the qualified accident reconstructionist.

Conservation of momentum, or the statement that the summation of momentum before minus the summation of momentum after the crash is zero, is not valid for low speed impacts with braking. For low speed crashes the difference in momentum before and after equals the external tire force impulse, computed by the product of braking force and crash time. If the tire braking forces are negligible, then the low impact speed analysis becomes considerably simpler, as discussed in the next section.

As we see when discussing low speed impact with braking, MARC 1 W4 may in some cases prove superior to systems using the coefficient of restitution only in low speed collisions without braking. The reason is that input data more easily obtained such as static crush depth, crush energy, stiffness, and a check against maximum vehicle acceleration may be used as a guide in the selection of a proper coefficient of restitution.

2.0 LOW SPEED IMPACT WITHOUT BRAKING

All mathematical details of the inline impact analysis without considering braking forces are discussed in Section 33-1 of the Text. The difference in low-versus high-speed impacts is expressed in terms of the coefficient of restitution. Lower coefficients of restitution are generally related to high-speed impact, while higher values relate to impacts closer to low speed impact. However, as a discussion of test results will show later, low impact speed tests may be associated with coefficients of restitution as low as 0.25 to 0.3 (Reference 1).

Inline collisions without braking with restitution are analyzed by MARC 1 W2. Vehicle 1 strikes Vehicle 2 in a central impact without rotation. Using the nomenclature of the Text, the following equations can be derived for each vehicle.

The velocity changes or delta-Vs of the vehicles are computed by:

$$\text{Vehicle 1: } \Delta V_1 = -m_2(1 + e)(V_{11} - V_{21}) / (m_1 + m_2), \text{ ft/sec} \quad (1)$$

$$\text{Vehicle 2: } \Delta V_2 = m_1(1 + e)(V_{11} - V_{21}) / (m_1 + m_2), \text{ ft/sec} \quad (2)$$

The coefficient of restitution e is defined by the ratio of relative velocities after to the relative velocities before impact:

$$e = (V_{22} - V_{12}) / (V_{11} - V_{21}) \tag{3}$$

The authors of Reference 2 conducted low impact speed tests of a 1993 Volvo 850 against a rigid barrier. MARC 1 W2 was used to compare test data with theory. To analyze impact tests against a rigid barrier, the weight of vehicle 2 was made very large as shown in Figure 1. The measured test results and MARC 1 W2 calculations show the following:

Impact Speed	Delta-V	Coefficient of Restitution	Delta-V MARC1 W2
5.75 mph	- 8.02 mph	0.39	- 8.00 mph
6.97 mph	- 9.54 mph	0.37	- 9.55 mph
12.98 mph	- 16.67 mph	0.28	- 16.61 mph

The correlation is excellent for two basic reasons. Standard inline collisions without braking involve no difficult mathematical formulation, and the coefficient of restitution was measured in the crash test.

Under normal circumstances an expert, asked to reconstruct an accident involving a single vehicle low speed impact against a solid wall, would not have been able to simply “guess” the correct coefficient of restitution. Depending upon the particular vehicle involved, an airbag may or may not have deployed for crashes involving speeds between 6 and 13 mph. The expert can use MARC 1 to run “crash tests on paper” to determine the delta-V sensitivity to the coefficient of restitution. For example, using $e = 0.7$ in the 5.75 mph test yields a delta-V = -9.79 mph. The use of an incorrect coefficient of restitution (0.7) may easily cause an expert to draw a wrong conclusion, namely that the airbag should have deployed. The reader is referred to Chapter 40 of the supplement of the Text for detailed information for conditions on airbag deployment. Although the damage shown in the photographs may appear extensive, no airbags deployed.

In the reconstruction of low impact speed accidents the coefficients of restitution are frequently difficult to determine. Test results and the analysis of real accidents have shown that the e-values rarely exceed 0.6. Reference 1 provides coefficient of restitution values for a number of low impact speed inline front bumper-to-rear bumper collisions between passenger cars. For example, when a Chevrolet Cavalier (W = 2320 lb) rear-ended a stationary Chevrolet Celebrity (W = 2754 lb) at 1.79 mph (0.8 m/s), the Celebrity experienced a delta-V = 1.34 mph (0.6 m/s). The coefficient of restitution was $e = 0.73$, based upon the measured velocity data. However, when the Cavalier rear-ended the Celebrity at 12.98 mph (5.8 m/s), the coefficient of restitution was $e = 0.24$. The energy absorbed by the some bumper systems may be “lost” as in a suspension shock absorber, or may be given back as in a spring. Any expert will be in a difficult situation when testifying to the use of a specific coefficient of restitution without proper and verifiable data collection.

Reference 3 provides detailed information on low speed testing of energy absorbing bumpers (1981 to 1983 Ford Escorts) and associated coefficients of restitution. The tests

Friday, August 11, 2006

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS
***** PROGRAM 'W-2' RUN FOR Volvo Barrier Test *****
TWO VEHICLE IN-LINE COLLISION

	1993	YY
Information For Vehicles	VOLVO	Barrier
	850	xx
Before Impact Speed of Vehicle, MPH: ==>	6.97	0.00
Vehicle Weight, LBS: ==>	3150.00	9999999.00
Coefficient of Restitution, D'LESS: ==>	0.37	

After Impact Speed of Vehicle, MPH: ==>	-2.58	0.00

Delta-V for Vehicle 1, MPH: ==>	-9.55	
Delta-V for Vehicle 2, MPH: ==>	0.00	
Crush Energy, FT·LBS: ==>	4406.39	

Reference 2: V11 = 6.97 mph; delta-V1 = -9.54 mph; delta-V2 = 0 mph; V12 = -2.57 mph

Figure 1. Volvo Barrier Impact at 6.97 mph

were conducted at 5 mph, 10 mph, and 15 mph. For each speed a second test was conducted. One bumper under-ride test was conducted at 10 mph. Acceleration, velocity, and target bumper displacement were measured as a function of time. Coefficients of restitution for post-impact velocities peak values and at 100 ms were calculated. The peak value-based coefficients of restitution ranged from 0.39 to 0.14 (low to high speed), and 0.19 to 0.12 for the 100 ms based values.

The coefficients of restitution used in accident reconstruction of low impact speed cases must be researched carefully. It often is a complicated matter, depending upon specific bumper design, exact points of impact on the vehicles including under-ride, and mechanical condition of the bumper structure and related components of older vehicles. Rust and prior collisions may significantly affect the dynamic response during a low speed crash. While for moderate to high-speed impacts the coefficient of restitution is more predictable due to the absence of second order factors, this is not the case in low speed impacts. Material properties, localized structural details, that is, where and how on the front or rear of a car the contact occurs, non-linear “spring stiffness”, vehicle pitching and energy absorbed by shock absorbers all will affect the after-impact velocity of the struck vehicle, and hence, the coefficient of restitution.

In some cases involving newer vehicles, electronic data recorders (EDR) may provide delta-V data. The reader is referred to Chapter 43 of the Supplement of the Text for details involving a low speed impact of a Chevrolet Tracker against a guardrail. The higher the coefficient of restitution is, the higher the delta-V of the struck vehicle will be. The delta-V of any analysis, no matter how complicated, can never exceed the delta-V associated with a theoretical elastic impact ($e = 1$). In the limit, the delta-V of the struck vehicle can never be greater than twice the impact speed of the striking vehicle. For details see Section 33-1(d) of the Text.

3.0 LOW IMPACT SPEED WITH BRAKING

A review of the published literature reveals very little about any low impact speed testing with braking. Reference 4 discusses some extremely low impact speed testing with braking. We will refer to it later and see how accurate MARC 1 W4 is in duplicating test data with braking.

We will use published data of low speed two-vehicle rear-end crash testing without braking to determine the accuracy of MARC 1 for low impact speed crashes without braking, and to study the effects of braking by use of the MARC 1 W4 in low speed crashes. This methodology addresses three aspects of a low impact speed analysis:

1. Provides the reader with proper input data and how to obtain them.
2. How accurate is the MARC 1 Low Speed Analysis?
 1. Shows the general effect of braking on the delta-V of the struck vehicle.

The authors of References 1,2 and 3 present test data obtained in rear-end collisions without braking during the crash. Consequently, our discussion will focus on rear-end

low impact speed crashes. The reader is encouraged to obtain a copy of Reference 2, which shows photographs of vehicle damage and data output traces as a function of time.

3.1 INPUT DATA FOR LOW SPEED INLINE IMPACT ANALYSIS

The mathematical details of the low impact speed analysis with braking are discussed in the Supplement to the Text. Since some readers may be new to the detailed analysis of low speed impacts, a quick review of the basics of MARC 1 “Low Speed Inline Impact” is presented.

Three basic fundamental physical laws applying to the crash are involved, namely conservation of energy, Newton’s Second Law, and material properties through the coefficient of restitution.

Conservation of energy requires that the kinetic energy after impact equals the kinetic energy before impact minus energies dissipated due to crush damage and due to braking. Newton’s Second Law with braking states that the impulse due to external braking forces equals the difference in momentum before and after impact. Material properties are stated by the ratio of difference in velocity after to difference in velocity before which is simply a definition of the coefficient of restitution. Vehicle velocity changes during the crash phase are formulated by use of the appropriate velocity-time diagram (Section 20-1 of the Text). The result is a set of equations with eight unknowns, assuming that the velocity of struck vehicle at the moment of impact is known.

Since braking impulse is computed from the product of vehicle mass, deceleration and crash time, we must compute compression and restitution times for each vehicle. The times are computed from the dynamic crush (computed for each vehicle) and static crush (measured for each vehicle) and the maximum acceleration (computed for each vehicle). The maximum accelerations of each vehicle are computed from crush energy (EES values for each vehicle), the static crush depth (measured on each vehicle), and the braking deceleration specified as input data.

The braking energy dissipated during the collision is computed by the product of vehicle mass, deceleration, and distance traveled during the crash time. Consequently, distances traveled by each vehicle during the crash phase must be computed for each vehicle.

The program will crash when the relationship of input data chosen results in calculations involving the square root of a negative number. For example, the restitution time of Vehicle 1 is computed by the following equation (see Supplement to the Text):

$$t_{r1} = (3.14/2) ((S_{dyn1} - S_{st1})/a_{max1})^{1/2} \quad (4)$$

The dynamic crush depth is computed by:

$$S_{dyn1} = (m_1/k_1)(a_{max1} - a_1) \quad (5)$$

The maximum acceleration of the vehicle is computed by:

$$a_{\max 1} = (EES_1)^2/S_{st1} + a_1 \quad (6)$$

The input data for crush EES, stiffness k and static crush must be such that the dynamic crush always is greater than the static crush. Although that makes total sense, the user may be ignorant why the computer run is not working.

The computed total crash times for each vehicle (compression and restitution phase times) should be approximately the same.

Since the “spring force” (or reaction force) acting between both vehicles during impact is equal and opposite, the products of dynamic crush and stiffness of Vehicle 1 must be approximately equal to the product of dynamic and stiffness of Vehicle 2.

Making the weight of Vehicle 2 very large, for example, trying to model a crash with a rigid immovable object, causes the program to calculate unrealistically large values of impact speed for the impacting vehicle. This is caused by unrealistically large values of braking impulse and braking energy.

In Section 21-6(f) of the Text deformation and crush energy principles are discussed. The derivation of a simple crush energy equation (Equation 21-23) is presented in the Text.

Since the average crush or spring force multiplied by the average crush depth or spring force displacement equals the crush energy, dividing Equation 21-23 by average crush depth c yields an expression for the stiffness values k that may be used in MARC 1W4, provided the test data were obtained in low speed impact tests.

$$\text{Stiffness } k = (W/g)(b_0b_1 + (b_0)^2/(2c) + (1/2)(b_1^2/(2c))) ; \text{ lb/ft} \quad (7)$$

where: c = crush depth measured in feet.

Using Example 21-6 (page 294) of the Text yields the following spring stiffness for a crush depth of c = 0.2 ft:

$$\begin{aligned} k &= (2370/32.2)(3.6 \times 23.3/0.2 + (23.3)^2(0.2)/2 + (3.6)^2/(2 \times 0.2)) = \\ &= 12,555 \text{ lb/ft} \end{aligned}$$

A k-value = 12,556 lb/ft appears to be too small for low impact speed analyses. The reader is reminded that the b₀ and b₁ values used in Example 21-7 of the Text were obtained in moderate to high impact speed crash tests, and consequently, cannot be used directly for low impact speed analyses. For example, changing crash slope data to b₀ = 7 ft/sec and b₁ = 30 1/sec yields k = 24,657 lb/ft.

Reference 3 provides low impact speed test data on 1981 to 1983 Ford Escorts rear-end collisions on crush energy, and dynamic and static deformation of the target bumper. Some of the results are summarized below. We calculated the stiffness values by dividing crush energy by dynamic crush, or in this case, dynamic target bumper displacement as shown in the table that follows. The authors of Reference 3 conducted two sets of crash tests for each impact speed range with excellent data correlation achieved in each test group. One test was conducted where the front bumper of the bullet vehicle was lowered in order to under-ride the bumper of the target vehicle.

Test #	Test Speed mph	Energy Lost lbft	Dyn. Bumper Displacement ft	Dyn/Stat Crush	Stiffness lb/ft
2	4.72	877	0.025	1.50	35,080
4	9.85	4124	0.190	2.23	21,705
6	15.10	9277	0.279	2.02	34,107

As the ratio of dynamic to static crush/bumper displacement indicates, the ratio can achieve significant values for low impact speed crashes (Reference 5). Reference 5 provides stiffness values of 300 to 800 kN/m (20,500 to 55,000 lb/ft) in a sample calculation.

3.2. DODGE/SUZUKI REAR-END COLLISION

This test is described in Reference 2. A 1994 Dodge Grand Caravan impacted a stationary 1995 Suzuki Sidekick in two tests at 11.62 and 20.15 mph, respectively. The Sidekick had an externally mounted spare tire that significantly affected the dynamic response of the Sidekick when impacted by the hood in the 20 mph test.

We first transport all known data into MARC 1 W4 attempting to duplicate the crash test data shown in Reference 2. Since the coefficient of restitution was determined in the test, we choose $e = 0.617$ calculated from the test data (Reference 2 states $e = 0.62$ for an impact speed of 32.42 km/h). We are also guided in the proper use of input data by knowing that the maximum acceleration of the vehicles involved cannot achieve unrealistic values. Reference 2 provides acceleration traces from their crash tests, indicating maximum accelerations of approximately 165 and 260 ft/sec² for the Dodge and Sidekick, respectively. Equation (6) allows us to relate crush energy EES and static crush depth input data in some reasonable manner. The stiffness values must be chosen such the computed dynamic crush depth values are greater than the static crush depth values, however without being excessive. A recommended ratio of dynamic to static crush depth would be approximately 1.5 to 2.5.

The stiffness values k are measured in lb/ft. They are strongly dependent upon the particular contact area between the vehicles. A meaningful range may be 20,000 to 90,000 lb/ft depending upon the low speed accident under investigation. They must not be confused with the standard A and B stiffness coefficients.

Friday, August 11, 2006

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS
***** PROGRAM 'W-4' RUN FOR Dodge Caravan/Suzuki Sidekick *****
LOW SPEED INLINE REAR-END COLLISION WITH BRAKING

Information For Vehicles	VEHICLE 1 1994 DODGE Caravan	VEHICLE 2 1995 SUZUKI Sidekick
Vehicle Weight, LBS:	==> 3650.000	3297.000
Crush Energy Equivalent Speed, FT/SEC:	==> 7.300	6.000
Static Crush Depth, FT:	==> 0.250	0.150
Stiffness, LBS/FT:	==> 80000.000	90000.000
Deceleration During Impact, FT/SEC ² :	==> 0.000	0.000
Initial Speed Assumed for Vehicle 2, FT/SEC:	==> ---	0.000
Coefficient of Restitution, D'LESS:	==>	0.617
Maximum Acceleration at End of Compression Phase, FT/SEC ² :	==> 213.160	240.000
Dynamic Crush Depth, FT:	==> 0.302	0.273
Compression Phase Time, SEC:	==> 0.059	0.053
Restitution Phase Time, SEC:	==> 0.025	0.036
Crush Energy, LBS·FT:	==> 3020.318	1843.043
Impulses Due to Braking, LB·SEC:	==> 0.000	0.000
Speed of Vehicles After Impact, FT/SEC:	==> 3.974	14.516
Velocity Change of Vehicles, FT/SEC:	==> -13.112	14.516
Speed of Vehicles Before Impact, FT/SEC:	==> 17.086	0.000

Reference 2: V11 = 17 ft/sec; delta-V1 = - 13 ft/sec; delta-V2 = 14.5 ft/sec; amax1 = 161 ft/(secsec); amax2 = 258 ft/(secsec)

Figure 2. Dodge Caravan/Suzuki Sidekick at 12 mph

The MARC 1 W4 reconstruction without braking is shown in Figure 2. A crush energy of $EES_1 = 7.3$ ft/sec (4.98 mph) is equivalent to a crush energy of 3,020 lbft for the Dodge. The maximum accelerations computed are acceptable, particularly in view of the fact that the acceleration traces shown in Reference 2 are not always easily interpreted.

Inspection of Figure 2 reveals that good agreement has been achieved with reasonable variation of critical input data for crush energy and stiffness values. The reader is reminded, that the coefficient of restitution normally is not directly available, and must be obtained from published test data or the expert's own tests.

Although none of the low impact speed crash tests in Reference 2 involved braking, MARC 1 W4 reveals the influence of braking on ΔV_2 of the struck vehicle. Assuming that Driver 2 pushed the brake pedal hard, resulting in a deceleration of 30 ft/sec², the low speed impact data results are shown In Figure 3. In order for the Dodge to satisfy the same input data, and the effect of braking by the Sidekick, it travels at a slightly lower impact speed of 15.856 ft/sec (10.81 mph) versus 17 ft/sec (11.6 mph), while the ΔV_2 of the Sidekick is reduced to 12.12 ft/sec (8.26 mph) compared with 14.51 ft/sec (9.9 mph) without braking.

The MARC 1 W4 low impact speed analysis for the 20.1mph test of Reference 2 is shown in Figure 4. Good correlation is achieved for the test data without braking. The measured maximum acceleration values were again used to guide the selection of crush energy and crush depth values. The crush depth for the Dodge was estimated from the photographs in Reference 2. The stiffness values were reduced from those used in the 11.62 mph impact test to account for the softer hood crush on the Dodge, and unseen crush of the spare tire bracket of the Sidekick. Braking on part of Vehicle 2 at 30 ft/sec² yields a $\Delta V_2 = 21.14$ ft/sec (14.4 mph) compared with 16.2 mph without braking. When both vehicles are braking with 30 ft/sec² during the collision, the velocity change reduces to $\Delta V_2 = 20.56$ ft/sec (14.0 mph). Based upon this limited analysis, braking may reduce the ΔV of the struck vehicle by approximately 2 mph.

3.3. CHRYSLER LEBARON/DODGE PICKUP REAR-END COLLISION

The authors of Reference 2 conducted four low impact speed crash tests involving a Chrysler LeBaron against a stationary Dodge pickup equipped with a slide-in trailer hitch. The impact point due to the extending slide in hitch ball was concentrated in a narrow area. The damage photographs of the Chrysler allow a reasonable estimation of crush depths achieved during the different tests. In each test a nearly identical bullet vehicle Chrysler LeBaron was used.

The test data measured in the 12 mph impact test are analyzed by use of MARC1 W4 as illustrated in Figure 5. The acceleration data recorded during the crash test must be interpreted carefully depending upon how the hitch penetrated through the bumper structure during the different tests. For example, the 12 mph impact test produced a lower deceleration in the Chrysler than the 7.02 and 7.12 mph impact test.

Friday, August 11, 2006

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS
 ***** PROGRAM 'W-4' RUN FOR Dodge Caravan/Suzuki Sidekick *****
 LOW SPEED INLINE REAR-END COLLISION WITH BRAKING

	VEHICLE 1	VEHICLE 2
	1994	1995
	DODGE	SUZUKI
Information For Vehicles	Caravan	Sidekick
Vehicle Weight, LBS:	3650.000	3297.000
Crush Energy Equivalent Speed, FT/SEC:	7.300	6.000
Static Crush Depth, FT:	0.250	0.150
Stiffness, LBS/FT:	80000.000	90000.000
Deceleration During Impact, FT/SEC ² :	0.000	30.000
Initial Speed Assumed for Vehicle 2, FT/SEC:	---	0.000
Coefficient of Restitution, D'LESS:	==>	0.617
Maximum Acceleration at End of Compression Phase, FT/SEC ² :	213.160	210.000
Dynamic Crush Depth, FT:	0.302	0.273
Compression Phase Time, SEC:	0.059	0.057
Restitution Phase Time, SEC:	0.025	0.038
Crush Energy, LBS·FT:	3020.318	1843.043
Impulses Due to Braking, LB·SEC:	0.000	290.779
Speed of Vehicles After Impact, FT/SEC:	2.340	12.123
Velocity Change of Vehicles, FT/SEC:	-13.516	12.123
Speed of Vehicles Before Impact, FT/SEC:	15.856	0.000

Reference 2: V11 = 17 ft/sec; delta-V1 = - 13 ft/sec; delta-V2 = 14.5 ft/sec; amax1 = 161 ft/(secsec); amax2 = 258 ft/(secsec)

Figure 3. Dodge Caravan/Suzuki Sidekick with Braking

Friday, August 11, 2006

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS
 ***** PROGRAM 'W-4' RUN FOR Dodge Caravan/Suzuki Sidekick *****
 LOW SPEED INLINE REAR-END COLLISION WITH BRAKING

Information For Vehicles	VEHICLE 1	VEHICLE 2
	1994 DODGE Caravan	1995 SUZUKI Sidekick
Vehicle Weight, LBS:	==> 3650.000	3297.000
Crush Energy Equivalent Speed, FT/SEC:	==> 14.700	8.900
Static Crush Depth, FT:	==> 0.570	0.240
Stiffness, LBS/FT:	==> 50000.000	70000.000
Deceleration During Impact, FT/SEC ² :	==> 0.000	0.000
Initial Speed Assumed for Vehicle 2, FT/SEC:	==> ---	0.000
Coefficient of Restitution, D'LESS:	==>	0.519
Maximum Acceleration at End of Compression Phase, FT/SEC ² :	==> 379.105	330.042
Dynamic Crush Depth, FT:	==> 0.859	0.483
Compression Phase Time, SEC:	==> 0.075	0.060
Restitution Phase Time, SEC:	==> 0.043	0.043
Crush Energy, LBS·FT:	==> 12247.337	4055.208
Impulses Due to Braking, LB·SEC:	==> 0.000	0.000
Speed of Vehicles After Impact, FT/SEC:	==> 8.038	22.986
Velocity Change of Vehicles, FT/SEC:	==> -20.763	22.986
Speed of Vehicles Before Impact, FT/SEC:	==> 28.801	0.000

Reference 2: V11 = 29.5 ft/sec; delta-V1 = - 21 ft/sec;
 delta-V2 = 23.86 ft/sec; V12 = 8.54 ft/sec; amax1 = 320
 ft/(secsec); amax2 = 386 ft/(secsec)

Figure 4. Dodge Caravan/Suzuki Sidekick at 20 mph

Friday, August 11, 2006

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS
 ***** PROGRAM 'W-4' RUN FOR Pontiac/Dodge *****
 LOW SPEED INLINE REAR-END COLLISION WITH BRAKING

	VEHICLE 1	VEHICLE 2
	1989	1988
	CHRYSLER	DODGE
Information For Vehicles	LeBaron	Pickup 4x4
Vehicle Weight, LBS:	2750.000	5700.000
Crush Energy Equivalent Speed, FT/SEC:	10.500	6.900
Static Crush Depth, FT:	0.360	0.150
Stiffness, LBS/FT:	70000.000	90000.000
Deceleration During Impact, FT/SEC ² :	0.000	0.000
Initial Speed Assumed for Vehicle 2, FT/SEC:	---	0.000
Coefficient of Restitution, D'LESS:	==>	0.220
Maximum Acceleration at End of Compression Phase, FT/SEC ² :	306.250	317.400
Dynamic Crush Depth, FT:	0.374	0.624
Compression Phase Time, SEC:	0.055	0.070
Restitution Phase Time, SEC:	0.010	0.061
Crush Energy, LBS·FT:	4707.880	4213.929
Impulses Due to Braking, LB·SEC:	0.000	0.000
Speed of Vehicles After Impact, FT/SEC:	3.194	7.163
Velocity Change of Vehicles, FT/SEC:	-14.847	7.163
Speed of Vehicles Before Impact, FT/SEC:	18.041	0.000

Reference 2: V11 = 17.64 ft/sec; delta-V1 = - 13.81 ft/sec;
 delta-V2 = 7.62 ft/sec; V12 = 3.81 ft/sec; amax1 = 190
 ft/(secsec)

Figure 5. LeBaron/Dodge Pickup at 12 mph

Inspection of Figure 5 reveals reasonable agreement between measurement and MARC1-W4 reconstruction analysis. The crush depths of the Chrysler were estimated from photographs in Reference 2. Minor variations of critical input data most likely will achieve improved correlation between prediction and test data.

Braking by the Chrysler at 30 ft/sec^2 reduces its delta-V from 7.1 ft/sec (4.85 mph) to 3.77 ft/sec (2.57 mph).

3.4 PONTIAC BONNEVILLE/DODGE SHADOW REAR-END COLLISION

The author of Reference 4 presents reported test data of low speed impacts with and without braking where a Pontiac Bonneville impact a stationary Dodge Shadow.

The data for the 2.42 mph (1.08 m/s) crash test are illustrated in Figures 6 (MARC 1 W2) and Figure 7 (MARC 1 W4), both without braking. The measured test data are shown in the comment sections below the computer runs. The stiffness and crush energy values were estimated to match the test data. No damage photographs are shown in Reference 4. Inspection of Figure 7 indicates excellent agreement with the test results shown in Reference 4.

The test results obtained with braking for an impact speed of 2.42 mph are suspect, as the author indicates in Reference 4. We only attempted to achieve the known impact speed of 2.42 mph and the measured $\Delta V_1 = -2.43 \text{ ft/sec}$ of the striking vehicle (assuming that these data were most likely correct) by use of MARC 1 W4 and by reasonable variation of the input data as shown in Figure 7. We will also input the previously used coefficient of restitution $e = 0.228$, although the test results indicate the value to be $e = -0.62$. It appears that the influence of braking by the Dodge Shadow during the crash was such that it “messed up” some of the measured data on the struck vehicle.

The results of the MARC 1 W4 analysis with braking of 10 ft/sec^2 are shown in Figure 8. Both the impact speed of 3.54 ft/sec (3.589) as well as the velocity change of the striking vehicle of -2.43 ft/sec (-2.448) indicate good correlation with test results. Inspection of Figure 8 indicates a lower delta-V of the struck vehicle of 1.959 ft/sec as compared with 2.416 ft/sec without braking. No specific braking data were provided in Reference 4.

To show the effect of maximum braking by the struck vehicle, MARC1 W4 was used to match the impact speed of 2.42 mph with braking at 30 ft/sec^2 of the struck vehicle. The results illustrated in Figure 9 show that the ΔV_2 of the Dodge with heavy braking will decrease to 0.689 ft/sec (0.47 mph) when struck at 3.571 ft/sec (2.43 mph). Consequently, heavy braking of the struck reduced its delta-V from 2.416 ft/sec (no braking) to 0.689 ft/sec (with braking).

Friday, August 11, 2006

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS
***** PROGRAM 'W-2' RUN FOR Pontiac Bonneville/Dodge Shadow *****
TWO VEHICLE IN-LINE COLLISION

Information For Vehicles	XX CHRYSLER Bonneville	YY DODGE Shadow
Before Impact Speed of Vehicle, MPH: ==>	2.42	0.00
Vehicle Weight, LBS: ==>	3610.00	2870.00
Coefficient of Restitution, D'LESS: ==>	0.23	

After Impact Speed of Vehicle, MPH: ==>	1.10	1.66

Delta-V for Vehicle 1, MPH: ==>		-1.32
Delta-V for Vehicle 2, MPH: ==>		1.66
Crush Energy, FT-LBS: ==>		296.24

Reference 4: V11 = 2.417 mph; delta-V1 = -1.278 mph; delta-V2 = 1.70 mph

Figure 6. Pontiac/Dodge Shadow at 2.42 mph

Friday, August 11, 2006

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS
***** PROGRAM 'W-4' RUN FOR Bonneville/Dodge *****
LOW SPEED INLINE REAR-END COLLISION WITH BRAKING

Information For Vehicles		VEHICLE 1	VEHICLE 2
		xx PONTIAC Bonneville	yy DODGE Shadow
Vehicle Weight, LBS:	==>	3610.000	2837.000
Crush Energy Equivalent Speed, FT/SEC:	==>	1.900	1.400
Static Crush Depth, FT:	==>	0.050	0.050
Stiffness, LBS/FT:	==>	60000.000	60000.000
Deceleration During Impact, FT/SEC ² :	==>	0.000	0.000
Initial Speed Assumed for Vehicle 2, FT/SEC:	==>	---	0.000
Coefficient of Restitution, D'LESS:	==>	==>	0.228
Maximum Acceleration at End of Compression Phase, FT/SEC ² :	==>	72.200	39.200
Dynamic Crush Depth, FT:	==>	0.135	0.058
Compression Phase Time, SEC:	==>	0.068	0.060
Restitution Phase Time, SEC:	==>	0.054	0.022
Crush Energy, LBS·FT:	==>	202.362	86.343
Impulses Due to Braking, LB·SEC:	==>	0.000	0.000
Speed of Vehicles After Impact, FT/SEC:	==>	1.615	2.416
Velocity Change of Vehicles, FT/SEC:	==>	-1.899	2.416
Speed of Vehicles Before Impact, FT/SEC:	==>	3.514	0.000

Reference 4: V11 = 3.54 ft/sec; delta-V1 = - 1.870 ft/sec;
delta-V2 = 2.494 ft/sec

Figure 7. Pontiac/Dodge Shadow at 2.42 mph

Friday, August 11, 2006

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS
 ***** PROGRAM 'W-4' RUN FOR Bonneville/Shadow with Braking *****
 LOW SPEED INLINE REAR-END COLLISION WITH BRAKING

Information For Vehicles	VEHICLE 1	VEHICLE 2
	xx PONTIAC Bonneville	yy DODGE Shadow
Vehicle Weight, LBS:	==> 3610.000	2837.000
Crush Energy Equivalent Speed, FT/SEC:	==> 1.900	1.900
Static Crush Depth, FT:	==> 0.060	0.045
Stiffness, LBS/FT:	==> 60000.000	60000.000
Deceleration During Impact, FT/SEC ² :	==> 0.000	10.000
Initial Speed Assumed for Vehicle 2, FT/SEC:	==> ---	0.000
Coefficient of Restitution, D'LESS:	==> ---	0.228
Maximum Acceleration at End of Compression Phase, FT/SEC ² :	==> 60.167	70.222
Dynamic Crush Depth, FT:	==> 0.112	0.118
Compression Phase Time, SEC:	==> 0.068	0.064
Restitution Phase Time, SEC:	==> 0.046	0.051
Crush Energy, LBS·FT:	==> 202.362	159.031
Impulses Due to Braking, LB·SEC:	==> 0.000	101.245
Speed of Vehicles After Impact, FT/SEC:	==> 1.099	1.895
Velocity Change of Vehicles, FT/SEC:	==> -2.393	1.895
Speed of Vehicles Before Impact, FT/SEC:	==> 3.492	0.000

Reference 4: V11 = 3.54 ft/sec; delta-V1 = - 2.43 ft/sec;
 delta-V2 = 1.08 ft/sec

**Figure 8. Pontiac/Dodge Shadow at 2.42 mph with
 Medium Braking**

Friday, August 11, 2006

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS
 ***** PROGRAM 'W-4' RUN FOR Bonneville/Shadow with Braking *****
 LOW SPEED INLINE REAR-END COLLISION WITH BRAKING

Information For Vehicles	VEHICLE 1	VEHICLE 2
	XX PONTIAC Bonneville	YY DODGE Shadow
Vehicle Weight, LBS:	==> 3610.000	2837.000
Crush Energy Equivalent Speed, FT/SEC:	==> 2.700	2.000
Static Crush Depth, FT:	==> 0.060	0.045
Stiffness, LBS/FT:	==> 80000.000	60000.000
Deceleration During Impact, FT/SEC ² :	==> 0.000	30.000
Initial Speed Assumed for Vehicle 2, FT/SEC:	==> ---	0.000
Coefficient of Restitution, D'LESS:	==> ---	0.228
Maximum Acceleration at End of Compression Phase, FT/SEC ² :	==> 121.500	58.889
Dynamic Crush Depth, FT:	==> 0.170	0.131
Compression Phase Time, SEC:	==> 0.059	0.074
Restitution Phase Time, SEC:	==> 0.047	0.060
Crush Energy, LBS·FT:	==> 408.648	176.211
Impulses Due to Braking, LB·SEC:	==> 0.000	353.695
Speed of Vehicles After Impact, FT/SEC:	==> -0.125	0.689
Velocity Change of Vehicles, FT/SEC:	==> -3.696	0.689
Speed of Vehicles Before Impact, FT/SEC:	==> 3.571	0.000

**Figure 9. Pontiac/Dodge Shadow at 2.42 mph with
Maximum Braking**

4.0 CONCLUSIONS

- 1.0. MARC 1 W4, Low Speed Impact with Braking, correlates well with existing test data.
- 2.0. Low speed impacts without braking, in comparison to computer models using restitution only, are aided by MARC 1 W4 by assisting in the selection of a proper input data by cross-checking input data such as crush energy, static crush depth, stiffness, vehicle acceleration, crush depth ratios, compression and restitution times, and coefficient of restitution.
- 3.0. In low speed impacts the ratio of dynamic to static crush depth can be significant.
- 4.0. The coefficients of restitution determined in low speed impact tests can be unexpectedly low. Values as low as 0.2 were observed. Computer models relying solely on the coefficient of restitution for low impact speed calculations for a given crash are suspect.
- 5.0. In low speed impacts, involving speeds that may reach injury-causing velocity changes, heavy braking by the struck vehicle during the crash reduces the delta-V of the struck vehicle by up to 2.5 mph.

REFERENCES

1. A.L. Cipriani, et al.: "Low Speed Collinear Impact Severity: A Comparison between Full Scale Testing and Analytical Prediction Tools with Restitution Analysis, SAE 2001-01-0540.
2. C. Brian Tanner, et al.: "Coefficients of Restitution for Low and Moderate Speed Impacts with Non-Standard Impact Configurations", SAE 2001-01-0891
3. Thomas J. Szabo and Judson Welcher: "Dynamics of Low Speed Crash Tests with Energy Absorbing Bumpers", SAE 921573
4. Raymond M. Brach: Modeling of Low-Speed, Front-to-Rear Vehicle Impacts, SAE 2003-0100491
5. Werner Gratzner and Heins Burg: "Analyse von Serienkollisionen und Berechnung der Insassenbeschleunigung im gestossenen Fahrzeug", Verkehrsunfall und Fahrzeugtechnik, September 1994, Heft 9
6. David J. King, et al.: "Comparison Testing of Bumper Isolators", SAE 1999-01-0096
1. Andrew J. Happer, et al.: "Practical Analysis Methodology for Low Speed Vehicle Collisions Involving Vehicles with Modern Bumper Systems", SAE 2003-01-0492

2. M. Mastandrea and D. Vangi: "Influence of Braking Force in Low-Speed Vehicle Collisions", Proc. ImechE. Vol. 219 Part D: J. Automobile Engineering (Braking only by the striking vehicle).
3. Thomas J. Szabo: "Human Occupant Kinematic Response to Low Speed Rear-End Impacts", SAE 940532
4. Stefan Meyer: "Zur Belastung der Halswirbelsaeule bei Lkw-Pkw-Auffahrtskollisionen", Verkehrsunfall und Fahrzeugtechnik, November 1996, Heft 11
5. R.W. Thompson: "An Investigation Into Low Speed Rear Impacts of Automobiles", Master's Thesis of Applied Science, University of British Columbia, 1999. (A study using a pendulum simulating the striking vehicle, primarily focusing on injury mechanisms).
12. Clifford R. Tyner and Stuart D. Smith: "Low Speed Impact Crashes: Engineering Analysis", Dartmouth, NS, September 1998 (internet)