

# **MOTORCYCLE BRAKING DYNAMICS**

**By**

**Rudy Limpert, Ph.D.**

**PC-BRAKE, Inc.**

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**[www.pcbrakeinc.com](http://www.pcbrakeinc.com)**

## 1.0 INTRODUCTION

In recent issues of Accident Investigation Quarterly motorcycle braking systems as well as braking test data were discussed in detail (Ref. 1 and 2). The objectives of this article are to review theoretical aspects of straight-line level-road motorcycle braking, how to calculate braking forces, and to demonstrate the usefulness of the braking forces diagram in determining braking deceleration in actual braking cases.

## 2.0 DYNAMIC WHEEL NORMAL FORCES

The dynamic normal forces of the front and rear tire of a motorcycle as a function of deceleration are computed by expressions similar to those of the front and rear axle normal forces of cars (Ref. 3).

The dynamic normal forces are:

$$\text{Front:} \quad F_{zF} = (1 - \psi + \chi a)W; \text{ lb} \quad (1)$$

$$\text{Rear:} \quad F_{zR} = (\psi - \chi a)W; \text{ lb} \quad (2)$$

Where:  $\psi = F_{zRstatic}/W$

$$\chi = h_{cg}/L$$

$a$  = deceleration, g-units

$F_{zRstatic}$  = static rear wheel normal force, lb

$L$  = wheel base, in.

$h_{cg}$  = motorcycle center-of-gravity height, in.

$W$  = weight of motorcycle, lb

The term  $\chi a W$  represents the dynamic load transfer onto the front wheel (Equation 1), or off the rear wheel (Equation 2). For example, for a 0.7g stop,  $W = 700$  lb, and  $\chi = 0.5$ , the front wheel normal force increases by  $(0.5)(0.7)(700) = 245$  lb. Equations 1 and 2 are straight lines as a function of deceleration  $a$ .

For a detailed mathematical analysis of braking dynamics, it becomes convenient to express the normal forces of Equations 1 and 2 per unit weight, or:

$$\text{Front:} \quad F_{zF}/W = (1 - \psi + \chi a) \quad (1a)$$

$$\text{Rear:} \quad F_{zR}/W = (\psi - \chi a) \quad (2a)$$

## 3.0 TRACTION COEFFICIENT

When the operator applies control inputs to the brake system, the brake torque on the front or rear brake produces a braking force between tire and ground. The ratio of braking force to normal force existing between wheel and ground is defined as the

traction coefficient  $\mu_T$ . Traction coefficients can either be calculated by use of braking dynamics equations as it is done in this article, or determined from testing using transducer braking platforms frequently employed by vehicle manufacturers.

Consequently, the traction coefficients are:

$$\text{Front: } \mu_{TF} = F_{xF}/F_{zF} \quad (3)$$

$$\text{Rear: } \mu_{TR} = F_{xR}/F_{zR} \quad (4)$$

$F_{xF}$  (or  $F_{xR}$ ) = actual braking force on front (or rear) wheel produced between tire and ground as a function of operator brake control application force and brake system parameters, lb.

For example, if the operator were to apply the rear brake pedal such that  $F_{xR} = 112$  lb and  $F_{zR} = 348$  lb, then the traction coefficient on the rear wheel is  $\mu_{TR} = 112/348 = 0.32$ . For these braking conditions the rear wheel will not lock up for rear tire-road friction coefficients  $f_R > 0.32$ . However, for road conditions  $f_R < 0.32$  the rear brake will lock. The limit condition is achieved when  $\mu_{TR} = f_R$ .

#### 4.0. OPTIMUM BRAKING FORCES

When discussing optimum braking forces one must consider several limiting conditions. The word optimum (in contrast to the word ideal) reflects some constraints that are under the control of both the motorcycle manufacturer as well as the motorcycle operator. In this article I am only considering straight-line level-road braking. Consequently, braking-in-a-turn and braking- on-a-grade are not evaluated. However, the brake system design engineer may have considered particular aspects in choosing brake system components to yield optimum braking for both straight line and braking-in-a-turning.

Optimum braking in the absence of ABS brake systems generally involves two different aspects, namely shortest stopping distance or maximum deceleration. Achieving the shortest stopping distance requires of the operator to minimize brake application time while attempting to maintain the maximum deceleration possible. This braking condition is generally achieved by the operator "slamming" on the rear brake resulting in rapid lockup while modulating the front brake just below lockup. This type of braking maneuver will result in shorter stopping distances when the braking speed is below a certain critical value due to the pronounced affect of brake system application time on the overall stopping distance. Achieving maximum deceleration based upon tire-road friction available requires skillful modulation of both front and rear brake by the operator, generally requiring more application time. Consequently, optimum braking based upon tire-road friction may result in shorter stopping distances at high braking speeds where the influence of brake application times on over-all stopping distance is less.

Generally, optimum braking is defined in terms of the maximum wheels-unlocked deceleration for a specified tire road friction coefficient and that both brakes lock up or

ABS modulation occurs at the same instant, thus eliminating any operator influence. For simplicity, we further assume, that the tire-road friction coefficients are identical for front and rear tires, that is,  $f_F = f_R$ .

Consequently, the optimum braking condition can be stated as:

$$\mu_{TF} = \mu_{TR} = f_F = f_R = a \quad (5)$$

In different words, Equation 5 states, that the traction coefficients equal each other (meaning simultaneous lockup) and equal the tire road friction coefficient (meaning all the tire road friction is used for braking) and consequently, equal deceleration  $a$ .

Combining Equations 1 and 3, and solving for the actual front braking force yields:

$$F_{xF} = (1 - \psi + \chi a)W\mu_{TF}; \text{ lb} \quad (6)$$

Similarly, the rear braking force is:

$$F_{xR} = (\psi - \chi a)W\mu_{TR}; \text{ lb} \quad (7)$$

The optimum normalized braking forces of a motorcycle are obtained by using the optimum condition (Equation 5), that is, replacing  $\mu_{TF}$  and  $\mu_{TR}$  by deceleration  $a$ , resulting in parabolic curves:

$$\text{Optimum front braking force: } F_{xF}/W)_{\text{opt}} = (1 - \psi + \chi a)a \quad (8)$$

$$\text{Optimum rear braking force: } F_{xR}/W)_{\text{opt}} = (\psi - \chi a)a \quad (9)$$

Using MARC 1 VI- OPTIMUM BRAKING FORCES, the optimum braking forces for a motorcycle are shown in the computer output. The input data for the exemplar motorcycle case are:  $W = 740$  lb, static rear wheel force 481 lb, center-of-gravity height 28 in., wheelbase 4.6 ft. The corresponding dimensionless parameters of the motorcycle are:  $\psi = 481/740 = 0.65$  and  $\chi = 28/((4.6)(12)) = 0.507$ .

The decelerations in Equations 8 and 9 were varied from zero to 1.2g. Inspection of the results reveals that at  $a = 1.2g$  the normalized rear braking force  $F_{xR}/W = 0.05$ , that is, the rear braking force is nearly zero due to the fact that the rear wheel normal force is nearly zero (lifting off ground). Substitution into Equation 2 shows:

$$F_{zR} = (0.65 - (0.507)(1.2))(740) = 30.8 \text{ lb}$$

The optimum braking force curve is illustrated in Figure 1. The normalized front braking forces are plotted on the y-axis, the normalized rear braking forces on the x-axis. The inclined 45-degree lines are lines of constant deceleration  $a$ . On any point along a given line of constant deceleration the deceleration is constant. For example, for any point on the line connecting  $F_{xF}/W = 0.6 = F_{xR}/W$  the deceleration is constant with  $a = 0.6g$ .

Thursday, November 15, 2007

MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS  
\*\*\*\*\* PROGRAM 'V-1' RUN FOR MC Braking Dynamics \*\*\*\*\*  
OPTIMUM BRAKING FORCES

Information for Vehicle	1994 BMW xx
Vehicle Weight, LBS:	====> 740.00
Rear Axle Weight, LBS	====> 481.00
Vehicle Center of Gravity Height, IN:	====> 28.00
Vehicle Wheelbase, FT:	====> 4.60

DECELERATION	$F_{xF}/W$	$F_{xR}/W$
0.10 'g'UNITS	0.040	0.060
0.20 'g'UNITS	0.090	0.110
0.30 'g'UNITS	0.151	0.149
0.40 'g'UNITS	0.221	0.179
0.50 'g'UNITS	0.302	0.198
0.60 'g'UNITS	0.393	0.207
0.70 'g'UNITS	0.494	0.206
0.80 'g'UNITS	0.605	0.195
0.90 'g'UNITS	0.726	0.174
1.00 'g'UNITS	0.857	0.143
1.10 'g'UNITS	0.999	0.101
1.20 'g'UNITS	1.150	0.050

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MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS  
\*\*\*\*\* PROGRAM 'V-1' RUN FOR MC Braking Dynamics \*\*\*\*\*  
OPTIMUM BRAKING FORCES DIAGRAM

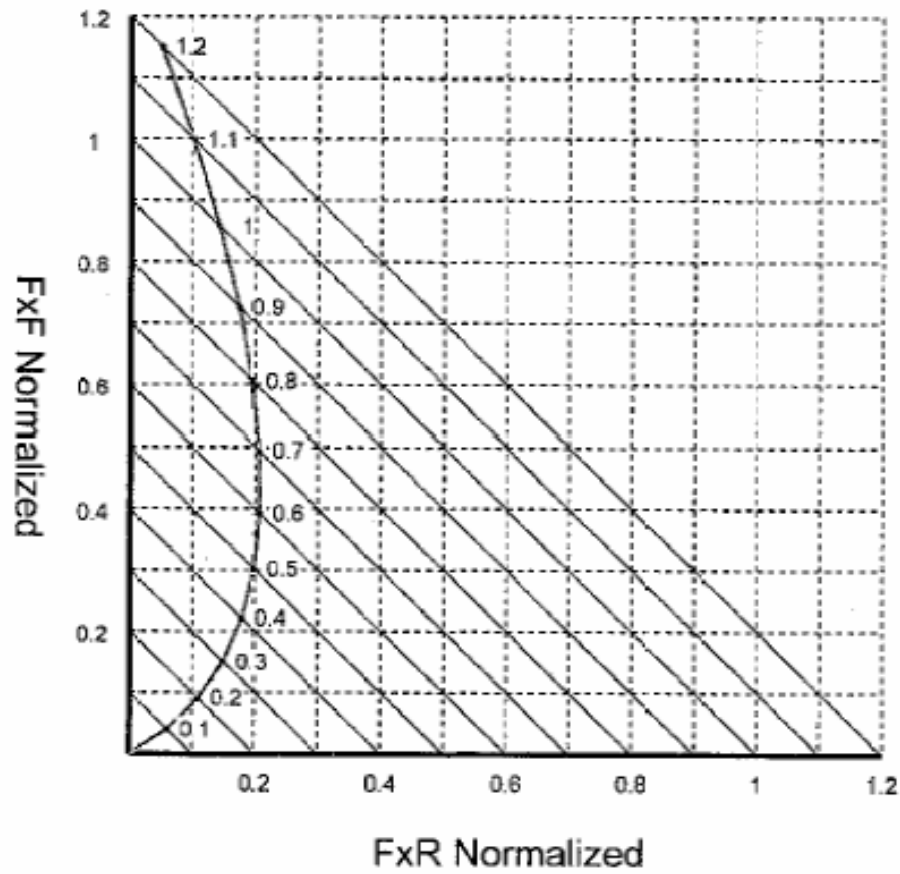


Figure 1 Optimum Braking Forces/  $\psi = 0.65$ ,  $\chi = 0.507$ ,  $W = 740$  lb

The optimum braking forces curve intersects each of the lines of constant deceleration. For example, the optimum curve intersects the line of constant deceleration  $a = 0.6g$  where  $F_{xF}/W = 0.4$  and  $F_{xR}/W = 0.2g$ . That this is correct can be shown easily from Newton's Second Law, namely (with deceleration expressed as  $a/g$ ):

$$F = Wa$$

or: 
$$F/W = F_{xF}/W + F_{xR}/W = a$$

or: 
$$0.4 + 0.2 = 0.6$$

Inspection of Figure 1 shows other pairs of front and rear normalized braking forces yielding  $a = 0.6g$ . For example,  $F_{xF}/W = 0.5$  and  $F_{xR}/W = 0.1$  also fall on the constant deceleration line of  $a = 0.6g$  line.

Any point located on the optimum curve identifies a pair of normalized front and rear braking forces that will result in optimum braking.

Inspection of Figure 1 also reveals that the rear braking force for the exemplar motorcycle case becomes zero when the deceleration exceeds  $1.2g$ . We can compute the exact deceleration from Equation 2a by setting  $F_{zR}/W = 0$ . Hence, we have:

$$F_{zR}/W = (\psi - \chi a) = 0$$

or: 
$$\psi = \chi a$$

or: 
$$a = \psi/\chi = 0.65/0.507 = 1.28g$$

The rear wheel of the motorcycle will lift off the ground at a deceleration of  $1.28g$ .

Equation 1a can be used to compute the acceleration (negative deceleration) required for the front wheel to lift off the ground by setting  $F_{zF}/W = 0$ . The result is:

$$a = (\psi - 1)/\chi = (0.65 - 1)/(0.507) = - 0.69g$$

With the motorcycle accelerating at  $0.69g$ , the front wheel will lift of the ground.

## 5.0. LINES OF CONSTANT TIRE-ROAD FRICTION COEFFICIENTS $f_{conF}$ and $f_{conR}$

As stated earlier, any point on the optimum braking forces curve in Figure 1 represents an optimum point. Consider point 0.8. At that point the tire-road friction coefficients on the front and rear tire are equal to 0.8, and both are equal to the deceleration  $a = 0.8g$ . The normalized braking forces are 0.6 for the front, and 0.2 for the rear. We now want to calculate the maximum front wheels-unlocked deceleration of the motorcycle when operating on a road with a tire-road friction coefficient of 0.8 and only the front brake is applied. Although this braking process may require a skilled operator or an ABS-equipped motorcycle, it constitutes limit braking performance.

We start with Equation 6 which determines the braking force produced on the front wheel. Obviously, since the rear brake is not applied, no optimum braking exists. According to Newton's Second Law, the braking force of the front wheel equals weight multiplied by deceleration  $a$ , that is,  $W_a$ , or:

$$F_{xF} = (1 - \psi + \chi a)W\mu_{TF} = Wa$$

Solving for deceleration  $a$ , yields:

$$a = (1 - \psi)\mu_{TF}/(1 - \chi\mu_{TF}); \text{ g-units} \quad (10)$$

Equation 10 is valid for any traction coefficient  $\mu_{TF}$ . However, as discussed earlier, when lockup occurs, the traction coefficient equals the tire road friction coefficient existing at the front wheel, that is,  $\mu_{TF} = f_F$ . Consequently, at the moment of front brake lockup or ABS modulation, we have:

$$a = (1 - \psi)f_F/(1 - \chi f_F); \text{ g-units} \quad (11)$$

Substituting the appropriate data of our exemplar motorcycle into Equation 11 yields the maximum front-braking-only deceleration of:

$$a_{\max} = (1 - 0.65)(0.8)/(1 - (0.507)(0.8)) = 0.471g$$

The motorcycle decelerates at  $a = 0.471g$  when braking with the front brake only on a roadway with a tire-road friction coefficient of  $f_F = 0.8$ . We can identify this data point on the  $F_{xF}/W$  axis (y-axis) of Figure 2. Connecting the optimum point equal to 0.8 located on the optimum curve with the newly established point located on the y-axis representing the deceleration of the motorcycle for the tire-road friction coefficient  $f_F = 0.8$  establishes the line of constant friction on the front wheel of  $f_{\text{conF}} = 0.8$ . The front wheel tire-road friction coefficient equals 0.8 anywhere on this straight line. The reason that this is correct derives from the two points through which the straight line is drawn, namely the optimum point for 0.8 and the front braking-only deceleration achieved on a roadway having a friction coefficient of  $f_F = 0.8$ .

Additional lines of constant front friction are obtained by using Equation 11 for different front wheel tire-road friction coefficients, say 0.1 through 1.2.

An equation similar to Equation 11 can be derived for rear braking-only, yielding data points on the x-axis to draw lines of constant rear tire road friction coefficients  $f_{\text{conR}}$ .

The result is:

$$a = \psi f_R/(1 + \chi f_R); \text{ g-units} \quad (12)$$

Substituting the appropriate data yields:

$$a = (0.65)(0.8)/(1 + (0.507)(0.8)) = 0.37g$$



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\*\*\*\*\* PROGRAM 'V-2' RUN FOR MC Braking Dynamics \*\*\*\*\*  
LINES OF CONSTANT FRICTION

Information for Vehicle	1994 BMW xx
Vehicle Weight, LBS:	====> 740.00
Rear Axle Weight, LBS:	====> 481.00
Vehicle Center of Gravity Height, IN:	====> 28.00
Vehicle Wheelbase, FT:	====> 4.60

TIRE-ROAD FRICTION	$a_F$	$a_R$
0.10 D'less	0.037	0.062
0.20 D'less	0.078	0.118
0.30 D'less	0.124	0.169
0.40 D'less	0.176	0.216
0.50 D'less	0.234	0.259
0.60 D'less	0.302	0.299
0.70 D'less	0.380	0.336
0.80 D'less	0.471	0.370
0.90 D'less	0.580	0.402
1.00 D'less	0.710	0.431
1.10 D'less	0.871	0.459
1.20 D'less	1.073	0.485

Consequently, the motorcycle decelerates at  $a_{max} = 0.37g$  on a roadway having a tire-road friction coefficient of 0.8 when only the rear brake is applied near or at lockup. Plotting the two data points 0.8 (optimum curve) and 0.37 (x-axis) and connecting them by a straight line yields the line of constant rear tire-road friction coefficient for  $f_{conR} = 0.8$  of the rear wheel.

The entire group of respective lines of constant tire-road friction coefficients front and rear with only the corresponding brake applied is shown in MARC 1 V 2 Lines of Constant Friction data print out. Their graphical illustration is shown in Figure 2. This braking forces diagram is the basic "work sheet" for the accident reconstructionist or motorcycle brake engineer. It is entirely based upon the geometrical and dynamic properties of the motorcycle in terms of wheelbase, horizontal weight distribution, and center-of-gravity height, that is, it is not a function of the brake system hardware in terms of wheel cylinder sizes, brake factors, or rotor radius, pedal ratios, that is, the installed braking system hardware. The braking forces diagram may be compared to the "DNA" of a motorcycle, since it is entirely unique for a specific motorcycle in terms of its values of  $\psi$  and  $\chi$ , that is, a particular loading condition.

## 6.0. ACTUAL BRAKING FORCES

In this section we will discuss the equations calculating actual braking forces developed by the brake system installed as a function of the operator's brake control inputs. The actual braking forces calculated are then compared with the optimum braking forces to determine decelerations for different braking and roadway conditions.

The brake factor BF of a brake is defined by the ratio of total drum (drum brake) or rotor (disc brake) drag  $F_d$  to the application force  $F_a$  of one shoe or one pad. Stated differently, the brake factor is the rotor friction drag force produced per one pound of shoe or pad application force, that is,

$$BF = F_d/F_a \quad (13)$$

For a disc brake,  $BF = 2 \mu_{pad}$ , where the pad-rotor friction coefficient is expressed by  $\mu_{pad}$ . Solving Equation 13 for rotor drag  $F_d$  yields:

$$F_d = F_a (BF); \text{ lb} \quad (14)$$

Equation 14 is the basic relationship for computing the braking force of a brake, and hence, braking effectiveness and deceleration of a motorcycle.

The braking force  $F_{xF}$  produced between the front tire and roadway as a function of front brake line pressure  $p_F$  is computed by:

$$F_{xF} = (p_F - p_{F0})(A_{wcF})(\eta_{wc})(BF_F)(n_F)(r_F/R_F); \text{ lb} \quad (15)$$

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MOTOR VEHICLE ACCIDENT RECONSTRUCTION AND CAUSE ANALYSIS  
\*\*\*\*\* PROGRAM 'V-2' RUN FOR MC Braking Dynamics \*\*\*\*\*  
LINES OF CONSTANT FRICTION DIAGRAM

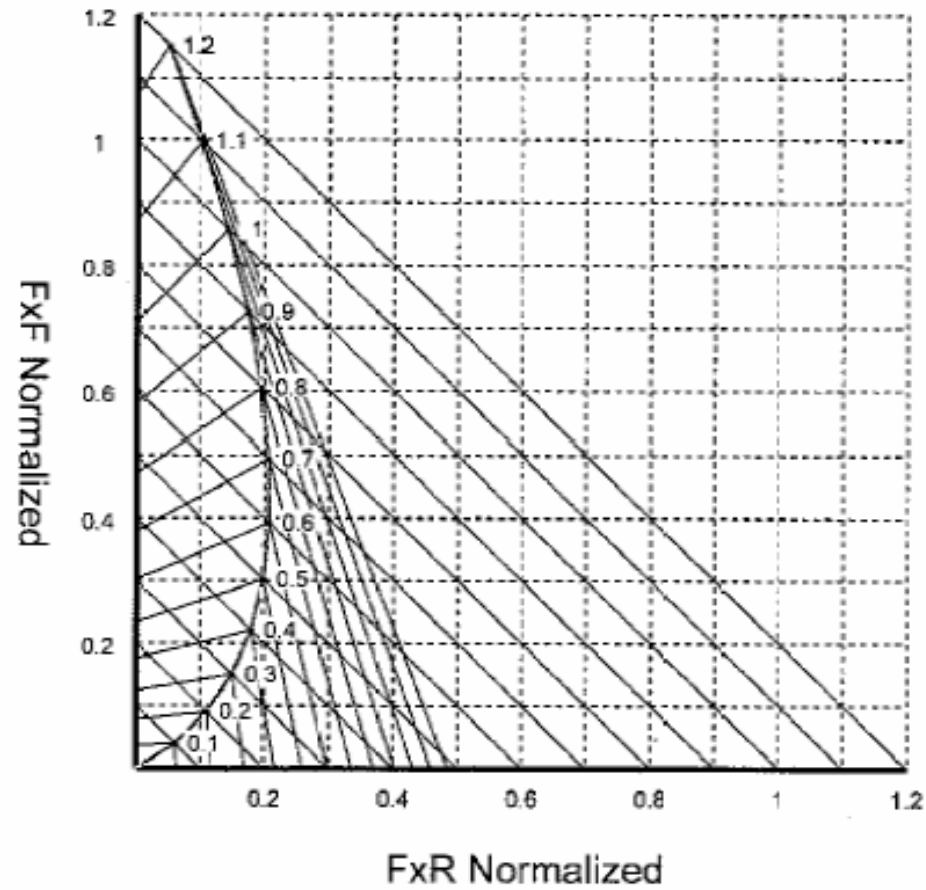


Figure 2 Braking Forces Diagram/  $\psi = 0.65$ ,  $\chi = 0.507$ ,  $W = 740$  lb

Where:

- $A_{wcF}$  = front wheel cylinder area, in.<sup>2</sup>
- $BF_F$  = front brake factor
- $n_F$  = number of brake rotors on front wheel
- $P_F$  = front hydraulic brake line pressure, psi
- $P_{Fo}$  = front brake push-out pressure, psi
- $r_F$  = front brake effective rotor radius, in.
- $R_F$  = front tire radius, in.
- $\eta_{wc}$  = efficiency of wheel cylinder

The front brake line pressure in case of an independent braking system is a function of the operator's hand application force and brake system parameters, such that

$$P_F = (F_h)(l_{mech})(\eta_l)/(A_{mcF}); \text{ psi} \quad (16)$$

Where:

- $A_{mcF}$  = front master cylinder cross-sectional area, in.<sup>2</sup>
- $F_h$  = operator's hand application force, lb
- $l_{mech}$  = mechanical gain between hand force brake lever and master cylinder input pushrod
- $\eta_l$  = efficiency of apply lever including master cylinder return spring

Expressions similar to Equations 15 and 16 can be developed for the rear brake of the motorcycle by replacing front brake subscripts in Equations 15 and 16 by the corresponding rear brake subscripts.

In case of an integrated brake system where pushing the rear brake pedal applies both the rear brake and one-half of the front brakes, special expressions can be developed to account for combined braking.

The deceleration  $a$  of a motorcycle can be computed from Newton's Second Law for a specified front apply force  $F_h$  and rear brake pedal force  $F_p$  as:

$$a = F_{xF}/W + F_{xR}/W; \text{ g-units} \quad (17)$$

MARC 1 V 3 Software was developed to calculate braking forces for automobiles or trucks equipped with hydraulic brakes where the ratio of front braking to rear braking (brake balance) is determined by the manufacturer, and therefore outside the control of the operator. For most motorcycles front-to-rear brake balance is determined by the operator's hand and/or foot force applied to the brake controls. For integrated brake systems the analysis presented earlier will be used to analyze the braking effectiveness of the motorcycle.

## 7.0. BRAKING FORCES DIAGRAM APPLICATIONS

In all applications we will use the same exemplar motorcycle data used in developing the braking forces diagram. The data are:  $W = 740$  lb,  $F_{zRstatic} = 481$  lb,  $h_{cg} = 28$  in.,  $L = 4.6$  ft. The "DNA" data of the example motorcycle are:  $\psi = 0.65$ ,  $\chi = 0.507$ . The tire-road

coefficients in our exemplar case are different front and rear. Due to excessive wear the front tire only produces  $f_F = 0.5$  at lockup or ABS modulation, while the rear tire produces maximum traction of  $f_R = 0.9$ .

### 7.1. REAR BRAKE ONLY

Equation 12 may be used to compute deceleration for a specified rear tire-road friction coefficient. In the braking forces diagram shown in Figure 3 the braking operating point (BOP) starts at the origin and moves along the  $F_{xR}/W$  axis until it reaches the line of constant rear tire-coefficient of friction  $f_{conR} = 0.9$  at point A. This cross-over point falls on the 0.4g line of constant deceleration. Checking against Equation 12 yields the same result of 0.4g as maximum deceleration at rear brake lockup or ABS modulation.

### 7.2. FRONT BRAKE ONLY

Consider Figure 3. With only the front brake applied, the BOP moves from the origin along the  $F_{xF}/W$  axis until it reaches the line of constant front tire coefficient of friction  $f_{conF} = 0.5$  at point B, a deceleration of approximately 0.23g. Checking with Equation 11 yields a deceleration  $a = 0.234g$ . The braking forces diagram shown on the computer screen of MARC 1 V2 can be enlarged to obtain an increased resolution for the diagram area of interest.

### 7.3. INDEPENDENT FRONT AND REAR BRAKES

We use the braking forces diagram shown in Figure 4 to determine the deceleration of the motorcycle at the moment when both the front and rear brake are locked or their ABS system modulates.

Assuming the driver applies the front brake first, the BOP reaches point B with a deceleration  $a = 0.234g$  when the front brake locks or ABS modulates. Now, while the motorcycle is decelerating at  $a = 0.234g$ , the operator applies the rear brake also with the normalized rear brake force  $F_{xR}/W$  moving along the x-axis to the right. However, the BOP moves along the front line of constant friction coefficient  $f_{conF} = 0.5$  until it reaches the optimum curve at point C. At this moment the normalized front braking force  $F_{xF}/W = 0.3$ , while  $F_{xR}/W = 0.2$ , yielding a deceleration  $a = 0.5g$ . The rear brake is not locked since the rear tire-road friction coefficient is 0.9 (and not 0.5). As the operator increases rear brake pedal force, the BOP moves along the front line of constant coefficient of friction  $f_{conF} = 0.5$  extended (manually) beyond point C until it reaches the rear line of constant coefficient of friction  $f_{conR} = 0.9$  at point D at a deceleration of approximately 0.63g when the rear brake locks or ABS modulation occurs.

The same result would have been achieved had the operator first applied the rear brake followed by front braking. When the operator applies both the front and rear brake simultaneously, the actual BOP is a direct function of how forcefully the operator applies both braking controls. Novice operators tend to apply the rear brake first and more fully, followed by a careful modulation of the front brake as illustrated by the heavy broken BOP line in Figure 4. Regardless of how the operator applies the braking controls, the

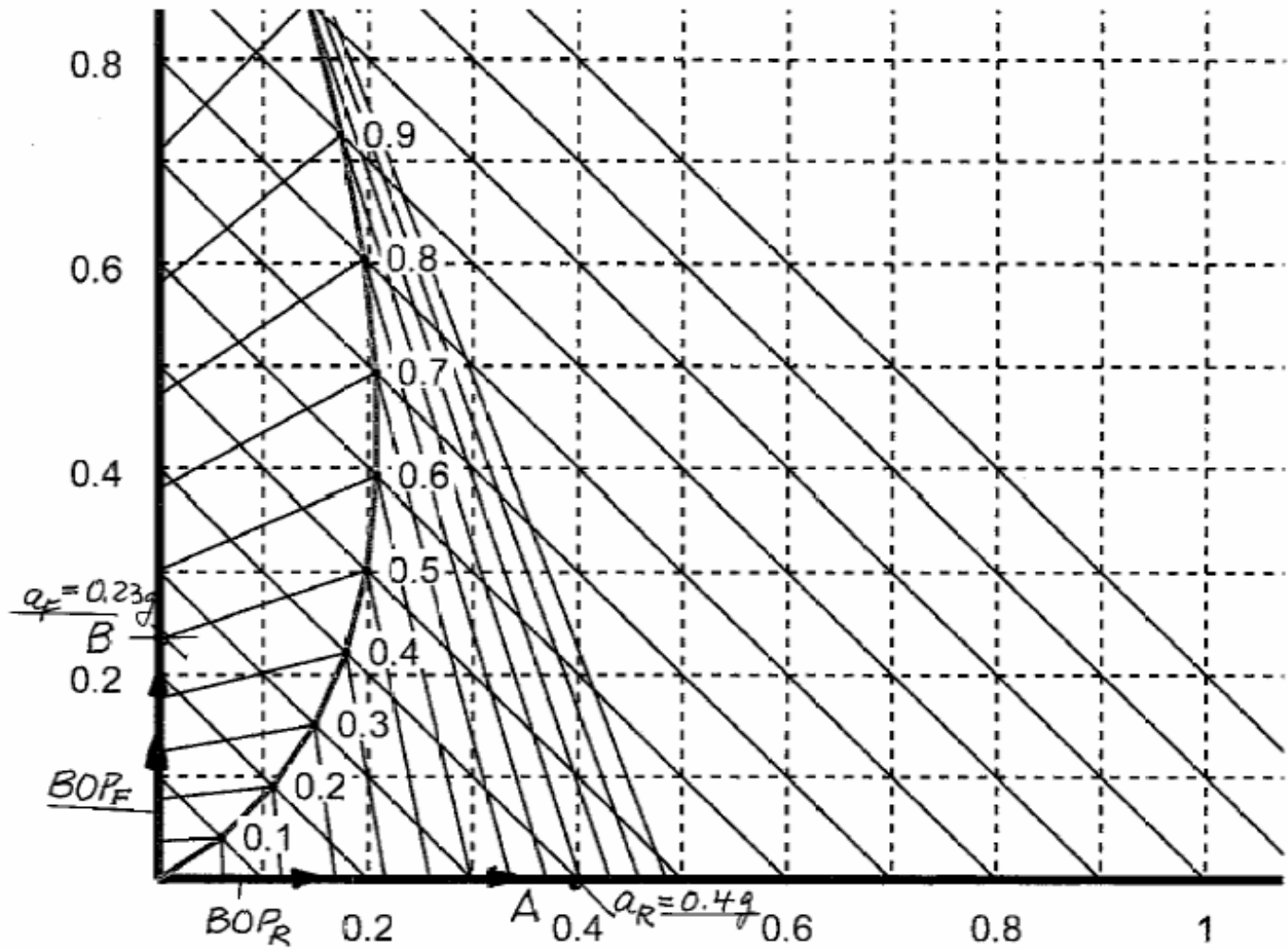


Figure 3 Rear and Front Brake Only,  $\psi = 0.65$ ,  $\chi = 0.507$ ,  $W = 740$  lb

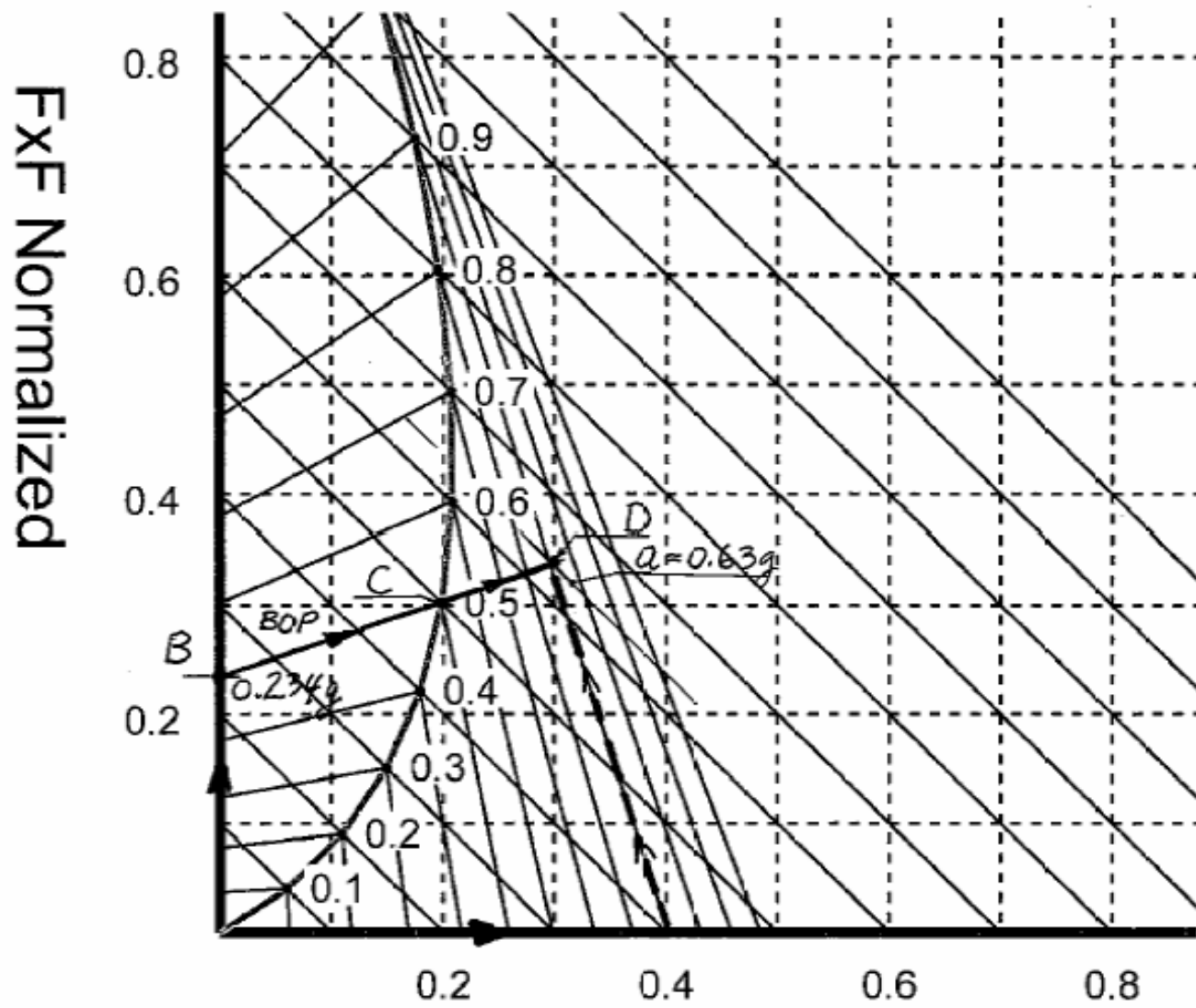


Figure 4 Independent Braking,  $\psi = 0.65$ ,  $\chi = 0.507$ ,  $W = 740$  lb

maximum deceleration of our exemplar motorcycle cannot exceed 0.63g due to its "DNA" and specified front and rear tire-road friction coefficients.

An empirical relationship has been developed (Ref. 4) where deceleration is expressed as a function of time to account for deceleration build-up time during the braking process:

$$a(t) = a_{\max}(1 - e^{-t/T}); \text{ g-units} \quad (18)$$

where:  $a_{\max}$  = maximum deceleration based upon vehicle and roadway parameters, g-units (0.63g in our exemplar case)  
 $T$  = time constant, sec (determined from experiments)  
 $t$  = time of braking, sec

$T$  is a function of operator skill level, braking speed, and maximum deceleration. For skilled operators  $T = 0.15\text{sec}$ , otherwise  $T = 0.3 \text{ sec}$ . Tests conducted with novice operators generally show sustained (not maximum based on tire road friction coefficient) decelerations approximately 40 to 50% lower than those achieved by skilled operators under similar conditions. The novice operators had motorcycle licenses for only four weeks (Ref. 4).

Applying Equation 18 to our exemplar case and assuming a novice operator, yields deceleration as a function of time:

t(sec)	0.25	0.50	0.75	1.00	2.00
a(t) (g-units)	0.36	0.51	0.58	0.61	0.63

Inspection of the "theoretical" numbers shows that a novice operator requires approximately two seconds to achieve the maximum braking effectiveness of 0.63g. Readers are reminded, that these are calculated results based upon test data without real-life accident threats. As in any operator-vehicle maneuver testing, it is one thing to determine what operators can do in a test under carefully established parameters, versus what real operators will do in a life-threatening accident avoidance maneuver never "practiced" before. In our exemplar motorcycle case the low front tire traction limiting maximum deceleration most likely will affect optimum operator response time.

The deceleration  $a = 0.63\text{g}$  achieved at point D can be computed from the front and rear lines of constant friction coefficients. At point D both lines intersect, and consequently, each line has the same  $F_{xF}/W$  and  $F_{xR}/W$  values. Following the rules of analytical geometry, the equations (of the form  $y = mx + b$ ) for the lines of constant friction can be derived as:

$$\text{Front: } F_{xF}/W = f_F\chi/(1 - f_F\chi)F_{xR}/W + (1 - \psi)f_F/(1 - f_F\chi) \quad (19)$$

$$\text{Rear: } F_{xF}/W = - (1 + f_R\chi)/(f_R\chi)F_{xR}/W + \psi/\chi \quad (20)$$

Solving Equations 19 and 20 simultaneously for  $F_{xF}/W$  and  $F_{xR}/W$  and substituting the exemplar motorcycle case data, as well as  $f_F = 0.5$  and  $f_R = 0.9$ , yields  $F_{xF}/W = 0.296$  and



$F_{xR}/W = 0.335$ , or  $a = 0.296 + 0.335 = 0.63g$  (Equation 17), indicating excellent agreement with our graphical solution shown in Figures 4 and 5.

#### 7.4. INTEGRATED BRAKING

For an integrated brake system application (usually) of the rear brake pedal applies one-half of the front brake (one brake rotor only instead of two) along with full braking of the rear brake.

The braking force distribution  $\Phi$  is defined by the ratio of rear braking force divided by the total braking force:

$$\Phi = F_{xR}/(F_{xF}+F_{xR}) \quad (21)$$

The braking forces are determined with Equation 15 applied for both front and rear brake. For our exemplar motorcycle case we assume that the front dual cylinder brake caliper for one rotor has a wheel cylinder piston diameter of 1 in., an effective rotor radius  $r_F = 6.1$  in., front tire radius  $R_F = 12$  in., a rear dual cylinder brake caliper wheel cylinder diameter of 1 in., an effective rear rotor radius of  $r_R = 5.08$  in., and rear tire radius  $R_R = 13$  in. All other brake components are assumed to be identical front and rear.

The front cross-sectional area on both wheel cylinders is  $(2)(1)^2(3.14)/(4) = 1.57$  in<sup>2</sup>. The cross-sectional area of the rear wheel cylinder area is  $(1)^2(3.14)/(4) = 0.785$  in<sup>2</sup>.

Substitution of the brake parameters usually different front to rear into Equation 21 yields

$$\Phi = (0.785)(5.08/13)/[(1.57)(6.1/12) + (0.785)(5.08/13)] = 0.278$$

The result indicates the basic brake system layout is such that 27.8% of the total braking force is concentrated on the rear wheel. Plotting  $F_{xR}/W = 0.278$  on the x-axis of Figure 5 (point A) and drawing a vertical line from there to the line of constant deceleration  $a = 1.0g$  yields point B. Drawing a horizontal line through B yields point C, the normalized front braking force  $F_{xF}/W = 0.722 = 1 - \Phi$ . The braking operating line for the integrated brake system is obtained by drawing a straight line from the origin to point B (unless hydraulic brake line pressure modulating valves are used).

Inspection of the braking forces diagram shows the following: The BOP moves from the origin along the inclined operating line until the BOP reaches the line of constant friction coefficient  $f_{conF} = 0.5$  at point D, where the front brake locks or the ABS system begins to modulate. The deceleration at point D is approximately  $a = 0.375g$ . If the driver continues to increase brake pedal force, the BOP moves along the line of constant coefficient of friction to the right until it reaches point E at a deceleration of approximately  $0.63g$  (same as in Section 7.3).

If in our exemplar motorcycle case both front and rear tire-road friction coefficients had been identical, say  $f_F = f_R = 0.8$ , inspection of the braking forces diagram shows that the BOP would move passed point D and the optimum line until it reaches point F, that is,

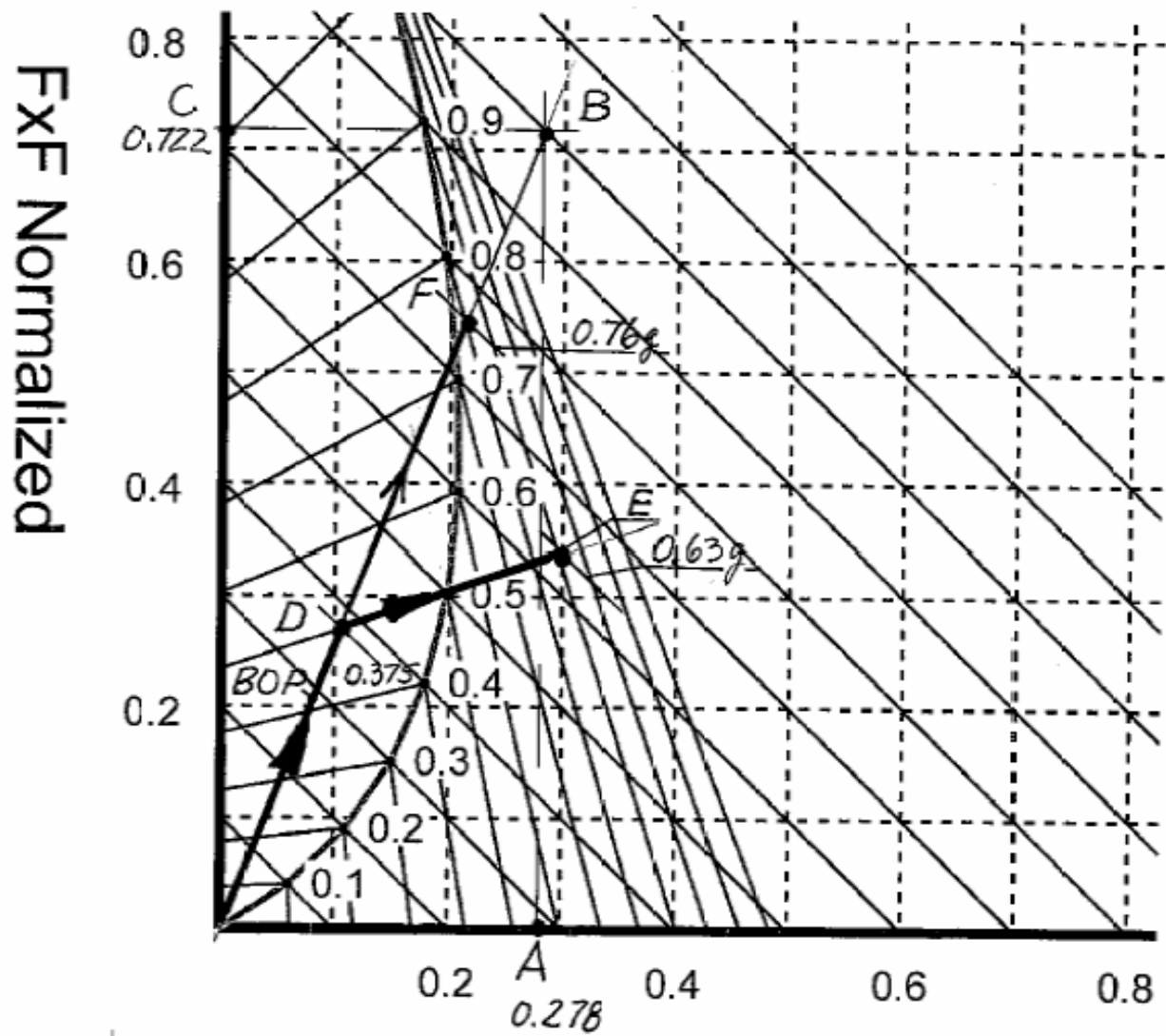


Figure 5 Integrated Braking System;  $\psi = 0.65$ ,  $\chi = 0.507$ ,  $\Phi = 0.278$

the rear line of constant friction coefficient at  $f_{\text{conR}} = 0.8$  at a deceleration of approximately 0.76g. The rear brake would lock before the front slightly below optimum conditions of  $a = 0.8g$ .

The exact deceleration at point D can be computed from the two straight lines intersecting at point D. One line is the line of constant rear friction coefficient  $f_{\text{conR}} = 0.8$  (Equation 20). The other line is the operating line of the form  $y = mx$  with its beginning at the origin, that is,  $b = 0$ . The slope  $m$  is determined from the brake balance as  $m = 0.722/0.278 = 2.597$ . Solving  $F_{\text{xF}}/W = 2.597 F_{\text{xR}}/W$  and Equation 20 simultaneously yields  $F_{\text{xF}}/W = 0.549$  and  $F_{\text{xR}}/W = 0.211$ , resulting in a deceleration of 0.76g.

## 8.0. OPERATOR ERRORS DURING EMERGENCY BRAKING

This section is not intended to provide a complete review of test data and accident statistical records. Its objectives are to present some information in reference to the braking analysis discussed. Emergency in this section has reference to the operator and accident avoidance braking maneuvers where maximum braking effectiveness or deceleration is required.

### 8.1. OPERATOR ERROR

The operator is not able to utilize the maximum tire-road friction available. Frequently, this error reveals itself when the operator applies the brakes hesitantly. In case of under-braking of the front brake the deceleration is significantly reduced. Lack of training and practice are often the reason. In many accidents involving motorcycle braking, the accident scene data show only a long rear tire brake mark, some times followed by a short front tire braking skid mark within a few feet of the point of impact. Even motorcycles equipped with ABS brakes are not fully utilized by inexperienced operators.

### 8.2. FRONT BRAKE LOCKUP PRIOR TO LOAD TRANSFER

In the entire braking analysis presented in this article the assumption was made the load transfer from the rear wheel onto the front wheel occurs instantly without any time delay. However, the increase in front wheel normal force experiences a time delay caused by the significant compression of the front springs. Consequently, immediate and full application of the front brake may result in lockup due to a lack of front wheel dynamic normal force. The danger of front brake lockup and capsizing of the motorcycle under these conditions are correspondingly great. ABS braking systems as well as special front suspensions equipped with kinematic pitch adjustment eliminate or minimize the effects of time delay.

## 9.0. BRAKE SYSTEM FACTORS AFFECTING BRAKING EFFECTIVENESS

### 9.1 FRONT BRAKE SHIMMY (WOBBLE)

Lateral run-out (LOR) greater than approximately 0.1 mm (0.0004 in.) of the front rotor causes the brake pad to rub against the high points of the disc when not braking, resulting

in non-uniform rotor thickness (disc thickness variation or DTV) around the circumference. When braking at moderate deceleration from higher speeds, DTV values greater than approximately 0.01 mm (0.00004 in.) will cause brake torque fluctuations resulting in hand brake lever oscillations as well as potential front end wobble.

## 9.2. BRAKE FADE

Brake fade is defined as a decrease in brake factor (brake torque) due a decrease in pad/rotor coefficient of friction caused by increased brake temperature. For motorcycle brakes initial fading may be a safety problem. In particular, the initial fading problem exists for "green" (new) organic pad materials which have not yet experienced a certain upper brake temperature limit. The binder component of the pad may change to a vapor, partially separating the pad from the swept surface of the rotor while braking. The result is temporary decrease of the pad friction coefficient and corresponding decrease of the brake factor (brake torque) and braking effectiveness. Once the binder is fully vaporized, the brake pad friction coefficient stabilizes at its design level.

## 9.3. BRAKE FLUID VAPORIZATION

Brake fluid vaporization occurs when the brake fluid in the front (or rear) caliper wheel cylinder exceeds the boiling temperature of the brake fluid. Brake fluid vapor assumes a significantly larger volume than liquid brake fluid, causing a compressible vapor pocket. The front (or rear) brake system cannot be pressurized effectively, resulting in partial or total front brake failure. The boiling temperature of brake fluid is a function of the type or quality of brake fluid used, and fluid contamination by water. Since brake fluid is hygroscopic it will absorb water naturally. Water will enter the brake system through the flexible brake hoses. Motorcycle brake maintenance must include regular brake fluid changes.

### References:

1. Accident Investigation Quarterly, Summer 2007
2. Accident Investigation Quarterly, Fall 2007
3. Brake Design and Safety, Rudolf Limpert, 2nd Edition, SAE International, 1999.
4. Unfallrekonstruktion, Volume 2, Wolfgang Hugemann, autorenteamGbR, 2007.

Note: The latest accident reconstruction software MARC 1 is available at no cost from the PC Brake website: [www.pcbrakeinc.com](http://www.pcbrakeinc.com) under link to MARC 1 – 2005.